

Supersymmetry beyond minimal flavour violation^{*}

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Abstract. We review the sources and phenomenology of non-minimal flavour violation in the MSSM. We discuss in some detail the most important theoretical and experimental constraints, as well as promising observables to look for supersymmetric effects at the LHC and in the future. We emphasize the sensitivity of flavour physics to the mechanism of supersymmetry breaking and to new degrees of freedom present at fundamental scales, such as the grand unification scale. We include a discussion of present data that may hint at departures from the Standard Model.

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1 Introduction

The Standard Model (SM) of particle physics has performed much better than could have been expected just after its establishment in the 1970s as a (conceptually) unified framework of all of particle physics based on the principle of quantum field theory with spontaneously broken gauge invariance: It has since then provided an economical description of thousands of measurements in particle physics and has so far eluded falsification. Nevertheless, there are known good reasons to believe in new physics, specifically supersymmetry (SUSY). Among them are:

- gauge coupling unification in the Minimal Supersymmetric Standard Model (MSSM), unlike the SM
- neutrino masses (and perhaps leptogenesis) from a seesaw mechanism at the GUT scale
- stabilization of the hierarchy $M_W \ll M_{\{\text{seesaw}, \text{GUT}, \text{PI}\}}$
- dark matter candidate, once proton stability is enforced by R -parity

If SUSY stabilizes the weak scale, we will know soon: the LHC has been designed to directly probe the TeV scale and clarify the mechanism of electroweak symmetry breaking. ATLAS and CMS should detect at least the lightest particle in the Higgs sector and likely part of the remaining superparticle spectrum directly. On the other hand, the examples given above demonstrate that precise measurements of low-energy observables can probe fundamental scales, including those beyond the “energy frontier”. This happens, for instance, if the fundamental physics violates accidental symmetries of the low-energy theory, i.e. lepton flavour as well as lepton number in the

case of neutrino oscillations in the context of the seesaw mechanism. Hadronic flavour transitions do occur in the SM, but they follow a very non-generic pattern as they are constrained by the $V - A$ structure of weak interactions and the absence of flavour-changing neutral currents (FCNC) at tree level. Moreover, they are suppressed by the scale of electroweak symmetry breaking, as well as the hierarchical structure of the Cabibbo-Kobayashi-Maskawa matrix which accounts for all flavour violation and all observed CP violation in the Standard Model. In consequence, there exist flavour-changing weak processes, such as neutral meson-antimeson mixing and certain weak decays, which are suppressed by multiple factors,

$$V_{ti}V_{tj}^* \times \frac{1}{16\pi^2} \times \frac{m^2}{M_W^2} \\ \sim 10^{-2} \times 10^{-2} \times 10^{-3} \quad \text{for FCNC } B_d \text{ decays,}$$

where m denotes the mass of the decaying particle. Comparing this to the corresponding factor $\frac{m^2}{M^2}$ due to a new flavoured particle with *generic* couplings and mass M , we see that even if $M \sim 10^{(3-4)} M_W \sim 10^2$ TeV, it can give rise to $\mathcal{O}(1)$ corrections to B -physics: Flavour physics is sensitive to very large scales. Indeed, such high sensitivity might seem too much: the absence of clear deviations from the SM pattern would seem to imply an unnaturally large scale of new physics (NP). Nevertheless, the absence of new flavoured degrees of freedom is unlikely to be an option in a theory of the weak scale, as the quadratic divergences of the Higgs/ W (mass)² are caused, in part, by fluctuations of a flavoured strongly interacting particle (the top). This guise of a “little hierarchy” is often called the new-physics flavour puzzle. Similar issues exist in the lepton sector, if new lepton-flavoured degrees of freedom are present (as in the MSSM). Of course, the flavour puzzle looks less severe for new physics that entails the same

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loop suppression as in the Standard Model. This is the case in the MSSM. Nevertheless, present flavour-physics data imposes strong constraints on the SUSY flavour structure. Finally, the data do show certain patterns that are consistent with deviations from the SM in “reasonable places”, i.e. NP-sensitive loop-dominated processes, albeit the significance is not (yet?) very high.

The remainder of this article is organized as follows. Section 2 deals with sources of SUSY flavour violation, the connection with supersymmetry breaking, the generic patterns that are expected depending on how SUSY is broken (and mediated), and constraints from internal consistency of the MSSM. Section 3 is devoted to the most important observables that either impose constraints on the MSSM flavour parameters at present or are likely to show signals in the future. We review and in some cases update bounds on “mass-insertion” parameters commonly used in the literature. There exist a number of articles devoted to this issue, foremost of all we would like to mention the original work of Gabbiani et al. [1] and the review article of Misiak, Pokorski, and Rosiek [2], on both of which we have drawn considerably. We next discuss, in Section 4, some correlation patterns that one may expect in certain SUSY GUTs, which illustrates the power to probe very high scales by combining information from different indirect observables. Finally, in Section 5 we mention several cases of observables where presently patterns of (mild) deviations from the SM are seen. All of them involve $b \rightarrow s$ transitions, which are precisely the domain of the LHCb experiment at CERN.

2 SUSY flavour violation

2.1 The unbroken MSSM is minimally flavour-violating

The MSSM (see [3,4,5] for reviews) stabilizes the weak scale by pairing bosons and fermions and relating their couplings to ensure a systematic Bose-Fermi cancellation, eliminating quadratic sensitivity to the cutoff (or scale of UV completion). In the MSSM, each chiral fermion is accompanied by a complex scalar “sfermion”, each gauge boson by a Weyl “gaugino”, and each of two higgs scalar(s) by “higgsinos” (see Table 1).

With new flavoured degrees of freedom, one generically expects modified flavour physics. However, the supersymmetrization itself does not introduce any new flavour structures. Indeed, in the limit where supersymmetry breaking is switched off, all flavour violation resides in the superpotential ($a \cdot b = a_1 b_2 - a_2 b_1$ is the invariant $SU(2)$ bilinear; our notation for the couplings conforms to the SUSY Les Houches accord conventions [6] wherever it overlaps):

$$W = \mu H_u \cdot H_d + Y_{ij}^U Q_i \cdot H_u U_j^c + Y_{ij}^D H_d \cdot Q_i D_j^c + Y_{ij}^E H_d \cdot L_i E_j^c. \quad (1)$$

Two higgs doublets H_u, H_d are required by gauge anomaly cancellation in the presence of the higgsinos. They are also necessary for fermion mass generation, as supersymmetry

Table 1. Supermultiplets in the MSSM (only propagating component fields are listed). Superpartners are denoted by a tilde. The last column denotes how often a multiplet appears.

multiplet	components of spin			SM gauge group representation	R -parity	#
	0	$\frac{1}{2}$	1			
Q	\tilde{q}	q		$(3, 2; 1/6)$	$-(-,+)$	3
U^c	\tilde{u}^c	u^c		$(\bar{3}, 1; -2/3)$	$-(-,+)$	3
D^c	\tilde{d}^c	d^c		$(\bar{3}, 1; 1/3)$	$-(-,+)$	3
L	\tilde{l}	l		$(1, 2; -1/2)$	$-(-,+)$	3
E^c	\tilde{e}^c	e^c		$(1, 1; 1)$	$-(-,+)$	3
H_d	h_d	\tilde{h}_d		$(1, 2; -1/2)$	$+(+,-)$	1
H_u	h_u	\tilde{h}_u		$(1, 2; 1/2)$	$+(+,-)$	1
V_G		\tilde{g}	G	$(8, 1; 0)$	$+(-,+)$	1
V_W		\tilde{w}	W	$(1, 3; 0)$	$+(-,+)$	1
V_B		\tilde{b}	B	$(1, 1; 0)$	$+(-,+)$	1

implies that each doublet has a well-defined (sign of the) hypercharge, hence can give mass to either $T_3 = +1/2$ or to $T_3 = -1/2$ SM fermions but not both. Thus in spite of the extra doublet, there are only three Yukawa terms, as many and as fundamental as in the SM. This relegates the origin of flavour breaking to more fundamental scales (an important difference to technicolour theories or extra-dimensional setups with bulk fermions).

Unlike in the SM, B and L are not accidental symmetries at the renormalizable level. In writing (1) we have omitted additional B or L -violating terms such as $U_i^c D_j^c D_k^c$ or $E_i^c L_j \cdot L_k$. The former, for example, generically mediates proton decay at unacceptably large rates. The extra terms are absent from (1) if R -parity is conserved. This also makes the lightest superpartner stable and restricts superpartners to only appear in loops in low-energy processes that involve only external SM particles.¹

The Lagrangian follows from the superpotential as

$$\mathcal{L} = \int d^4\theta K(\phi, \phi^*) + \left\{ \int d^2\theta W(\phi) + \text{h.c.} \right\} + \text{gauge kinetic terms.} \quad (2)$$

At the renormalizable level, the Kähler potential K is fixed to the form

$$K(\phi, \phi^*) = \sum_i \phi_i^* e^{2g_a V_a} \phi_i. \quad (3)$$

The interactions among the fermions and sfermions are then given in terms of W and gauge couplings as

$$\mathcal{L}_{\text{Yukawa}} = \frac{1}{2} \sum_{ij} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i^T C \psi_j + \text{h.c.}, \quad (4)$$

$$V(\{\phi_i\}) = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{1}{2} \sum_{aA} g_a^2 \left(\sum_i \phi_i^\dagger T_a^A \phi_i \right)^2, \quad (5)$$

¹ The behaviour of the fields under an R -parity transformation (reflection of superspace coordinates) is shown in the fourth column of Table 1. All “SM” particles (including both scalar higgs doublets) are even, all superpartners odd.

Table 2. D -term coefficients $d_{fLL,RR}$

	u	d	e
LL	$\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W$	$-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W$	$-\frac{1}{2} + \sin^2 \theta_W$
RR	$\frac{2}{3} \sin^2 \theta_W$	$-\frac{1}{3} \sin^2 \theta_W$	$-\sin^2 \theta_W$

and the fermion-sfermion-gaugino interactions are proportional to gauge couplings. This leaves the MSSM with one parameter *fewer* than the Standard Model. (The missing parameter is the quartic Higgs coupling, which is fixed in terms of gauge couplings.)

On the flavour side, the gauge interactions respect an $SU(3)^5$ flavour symmetry – separate transformations

$$\phi_i \rightarrow U_{ij}^\phi \phi_j \quad (\phi = Q, U^c, D^c, L, E^c) \quad (6)$$

among the three generations of any of the five types of irreducible supermultiplets – which is broken only by Y^U, Y^D, Y^E . Hence at this stage, the MSSM is minimally flavour violating [7,8,9,10]. Indeed, once electroweak symmetry is broken by vacuum expectation values

$$\langle h_u \rangle = \begin{pmatrix} 0 \\ \frac{v_u}{\sqrt{2}} \end{pmatrix}, \quad \langle h_d \rangle = \begin{pmatrix} \frac{v_d}{\sqrt{2}} \\ 0 \end{pmatrix} \quad (7)$$

(which are real for a suitable phase convention for h_u, h_d), the mass matrix of the down-type sfermions in the so-called super-CKM basis² is compactly expressed in terms of 3×3 blocks as³

$$\begin{aligned} \mathcal{M}_f^{2,\text{sym}} &\equiv \begin{pmatrix} \mathcal{M}_{\tilde{f}LL}^{2,\text{sym}} & \mathcal{M}_{\tilde{f}LR}^{2,\text{sym}} \\ (\mathcal{M}_{\tilde{f}LR}^{2,\text{sym}})^\dagger & \mathcal{M}_{\tilde{f}RR}^{2,\text{sym}} \end{pmatrix} \\ &\equiv \begin{pmatrix} m_f^2 + D_{fLL} & -\mu m_f x_f \\ -\mu^* m_f x_f & m_f^2 + D_{fRR} \end{pmatrix}. \end{aligned} \quad (8)$$

Here the label $f = u, d, e$ denotes up-type squarks, down-type squarks, and charged sleptons, respectively, m_u, m_d , and m_e are the corresponding 3×3 *fermion* mass matrices, and $x_u = \cot \beta \equiv v_d/v_u$ and $x_d = x_e = \tan \beta = v_u/v_d$. The D -term contributions $D_{fLL(RR)}$ are proportional to the 3×3 unit matrix, i.e., flavour-blind:

$$\begin{aligned} D_{fLL,RR} &= \cos(2\beta) m_Z^2 (T_{3f} - Q_f \sin^2 \theta_W) \mathbf{1} \\ &\equiv d_{fLL,RR} \cos(2\beta) m_Z^2 \mathbf{1}. \end{aligned} \quad (9)$$

The values $d_{fLL,RR}$ are reported in Table 2. Clearly, the sfermion mass matrices are diagonal in the super-CKM basis. In other words, the flavour sfermion states coincide with sfermion mass eigenstates. We note that the winos,

² The super-CKM basis (of superfields) [11,12] is defined by requiring diagonal Yukawa couplings in (1) and the CKM matrix being in its four-parameter standard form.

³ The order of rows and columns is: $(\tilde{d}_L, \tilde{s}_L, \tilde{b}_L, \tilde{d}_R, \tilde{s}_R, \tilde{b}_R)$, where $\tilde{d}_{iL} \equiv \tilde{d}_i$, $\tilde{d}_{iR} \equiv (\tilde{d}_i^c)^*$, for \mathcal{M}_d^2 , with obvious analogy for up-type squarks and charged sleptons.

binos, and higgsinos shown in the vertices are not mass eigenstates, but rather mix to form charginos and neutralinos. However, this does not affect the flavour structure of the vertices. The fact that sfermion mass and flavour states coincide implies that flavour violation is exclusively due to (s)fermion-charge-changing interactions. Those that involve fermions take the form of charged-wino-quark-squark, charged-higgsino-quark-squark, and charged-Higgs-quark-quark couplings, besides the Wud and Wlv couplings in the Standard Model (Fig. 1). We note that all flavour violation at vertices is governed in the obvious way by the CKM matrix. In the figure, we have switched to four-component fermion notation. Furthermore we have defined

$$\tilde{f}_R \equiv (\tilde{f}^c)^*. \quad (10)$$

The left-handed quark flavour states are related to the weak doublets (“interaction basis”) via

$$\tilde{q}_{Li} = (V_{ki}^* \tilde{u}_{Lk}, \tilde{d}_{Li}), \quad (11)$$

where, as is common, we have selected out of the infinite set of interaction bases one where the down-type Yukawa couplings are diagonal.⁴ The sneutrino flavour states are defined as $SU(2)$ and SUSY partners of the charged-lepton mass eigenstates,

$$\tilde{l}_{Li} = (\tilde{\nu}_{Li}, \tilde{e}_{Li}). \quad (12)$$

This minimal set of flavour-violating vertices immediately leads to a very non-generic pattern of (quark-) flavour transitions. In particular, $b \rightarrow s$, $b \rightarrow d$, and $s \rightarrow d$ transitions are correlated, as they are in the SM [9]. The nonminimal flavour violation that is the title of this article is then exclusively due to supersymmetry breaking. Strictly speaking, our title is an oxymoron: it should really read “Flavour violation beyond the SUSY limit”! This makes clear that the search for departures from minimal flavour violation provides a line of approach in unraveling the SUSY breaking mechanism.

2.1.1 $\tan \beta$ parameter

The parameter $\tan \beta = v_u/v_d$ introduced below (8) plays an important role in Higgs physics, as it affects both the mass spectrum and the trilinear couplings. The tree-level relation $m_b = y_b v_d/\sqrt{2}$ implies that $y_b \sim y_t \sim 1$ becomes possible for $\tan \beta = \mathcal{O}(50)$ (as in certain grand unified models that entail bottom-top Yukawa unification). The relevance to flavour physics derives from the fact that this can lead to double enhancement factors in front of loops of the form [13,14,15,16,17,18,19],

$$\tan \beta y_b \propto \tan^2 \beta m_b,$$

which can compensate the loop suppression in Higgs-mediated contributions to FCNC observables which are usually small

⁴ A model of flavour would single out a (at least one) different basis (in general) where the Yukawa matrices would look “simple”.

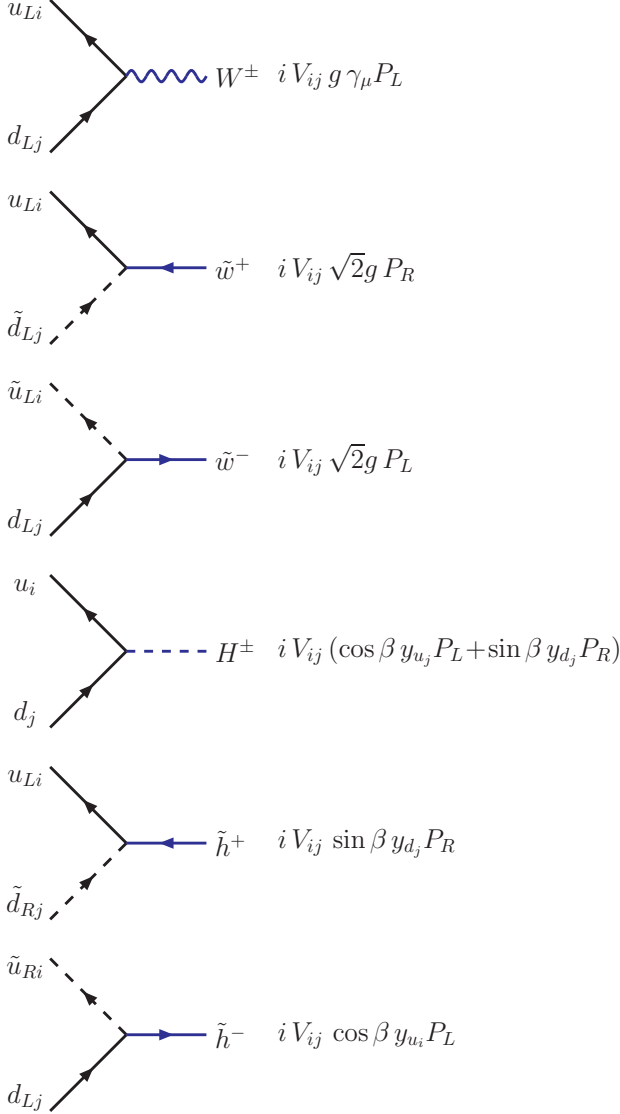


Fig. 1. Flavour-changing vertices involving fermions in the super-CKM basis.

for small to moderate (< 30) values of $\tan \beta$ but can give rise to a distinctive pattern at larger values even for minimal flavour violation. We will not discuss these effects; for a recent review see [20]. Most of the constraints discussed below still apply in that case, but there may be stronger ones.

2.2 Origin of (new) flavour violation: supersymmetry breaking

The superpotential (1) does not break supersymmetry spontaneously at tree level. Because of supersymmetric non-renormalization theorems [21, 22, 23], this remains true to all orders in perturbation theory. Neither is electroweak symmetry broken, at any order.

Observations exclude the presence of mass-degenerate superpartners for many of the SM particles, which tells us that supersymmetry is broken. The standard picture is that supersymmetry breaking occurs in a hidden sector of SM gauge singlets, via the condensation of an auxiliary (F or D) component of one or more superfields X . Gauge symmetry then requires any superpotential couplings between the visible and hidden sectors to be nonrenormalizable.⁵ In many cases of interest, all low-energy effects of supersymmetry breaking can be represented by such effective nonrenormalizable superpotential, gauge-kinetic, and Kähler terms, as in

$$W_{\text{break}} = \mathcal{A}_{ij}^U \frac{\langle X \rangle}{M} U_i^C H_u \cdot Q_j, \quad (13)$$

$$f_{\text{break}} = \mathcal{M}_a \frac{\langle X \rangle}{M} W_a^A W_a^A, \quad (14)$$

and

$$K_{\text{break}} = \mathcal{K}_{ij}^Q \frac{\langle X X^\dagger \rangle}{M^2} Q_i^\dagger e^{2g_a V_a} Q_j. \quad (15)$$

Here \mathcal{A}_{ij}^U , \mathcal{M}_a , and \mathcal{K}_{ij}^Q are dimensionless coefficients. $\langle X \rangle = \theta^2 F_X$ is the vacuum expectation value of a hidden-sector superfield, and the SUSY-breaking terms in the Lagrangian are found by replacing $K \rightarrow K + K_{\text{break}}$ and $W \rightarrow W + W_{\text{break}} + f_{\text{break}}$ in (2). This can be illustrated as follows. The MSSM, by assumption, does not have any direct renormalizable couplings to the hidden sector. Assume then that the lightest “messenger”, i.e., degree of freedom that couples both to the field X and to the MSSM fields, has mass M . Below its mass scale, it can be integrated out of the theory, giving rise to operators as in (13)–(15). This is what happens, for example, in models of gauge mediation (see below).

The term W_{break} from above gives rise to an extra contribution

$$\Delta \mathcal{L}_A = T_{ij}^U \tilde{q}_i \cdot h_u \tilde{u}_j^c + \text{h.c.},$$

$$T_{ij}^U = \frac{F_X}{M} \mathcal{A}_{ij}^U \quad (16)$$

⁵ The one exception is a possible coupling $H_u \cdot H_d X$, without imposing further global symmetries.

to the trilinear scalar coupling (traditionally called “A-term”). On the other hand, (14) generates gaugino masses, while (15) gives rise to extra contributions

$$\begin{aligned}\Delta\mathcal{L}_{m^2} &= m_{\tilde{q}ij}^2 \tilde{q}_i^\dagger \tilde{q}_j, \\ m_{\tilde{q}ij}^2 &= \frac{F_X^2}{M^2} \mathcal{K}_{ij}^Q\end{aligned}\quad (17)$$

to the masses of the sfermions (relative to those of their fermionic partners).

The crucial point for flavour is the following: Barring any supplement of the MSSM by a theory of flavour, naturalness dictates that the flavour structures of $m_{\tilde{q}ij}^2$ and T^U should be assumed generic numbers $\mathcal{O}(1)$ and, in particular, independent of Y^U . Hence the SUSY-breaking, renormalizable (masses and) interactions among the sfermions provide a generically nonminimal source of flavour violation. What can one say about the mass scale M ? First, (17) shows that the effective SUSY-breaking mass scale is set by

$$m_{\text{SUSY}} = F_X/M$$

which should be of $\mathcal{O}(\text{TeV})$ to preserve SUSY as a solution to the hierarchy problem, thus M can in principle range from not far beyond the SUSY scale. On the other hand, any global flavour symmetries which would forbid or restrict the nonrenormalizable terms of the gauge sector are expected to be broken (nonminimally) by gravitational physics, such that $M < M_{Pl}$.

Generalizing (13) and (15) and including operators as in (14) that give rise to gaugino masses induces the following set of dimensionful SUSY-breaking terms:

$$\begin{aligned}\mathcal{L}_{\text{soft}} &= -m_{\tilde{q}ij}^2 \tilde{q}_i^\dagger \tilde{q}_j - m_{\tilde{u}ij}^2 \tilde{u}^{c\dagger} \tilde{u}^c - m_{\tilde{d}ij}^2 \tilde{d}^{c\dagger} \tilde{d}^c \\ &\quad - m_{\tilde{l}ij}^2 \tilde{l}^\dagger \tilde{l} - m_{\tilde{e}ij}^2 \tilde{e}^{c\dagger} \tilde{e}^c - m_{h_u}^2 h_u^\dagger h_u - m_{h_d}^2 h_d^\dagger h_d \\ &\quad - \left[m_1 \tilde{b} \tilde{b} + m_2 \tilde{w}^A \tilde{w}^A + m_3 \tilde{g}^A \tilde{g}^A + B_\mu h_u \cdot h_d \right. \\ &\quad \left. + T_{ij}^U \tilde{q}_i \cdot h_u \tilde{u}_j^c + T_{ij}^D h_d \cdot \tilde{q}_i \tilde{d}_j^c + T_{ij}^E h_d \cdot \tilde{l}_i \tilde{e}_j^c + \text{h.c.} \right].\end{aligned}\quad (18)$$

$\mathcal{L}_{\text{soft}}$ does not introduce any quadratic divergences. In particular, the soft-breaking terms themselves are only logarithmically sensitive to heavy scales such as the seesaw scale.

From a purely phenomenological point of view, the supersymmetry breaking can simply be introduced explicitly according to the structure of $\mathcal{L}_{\text{soft}}$. In this case, the hierarchy is stabilized but not explained: all dimensionful terms in $\mathcal{L}_{\text{soft}}$ have to simply happen to be of order the weak scale. Indeed, often the MSSM is defined as the supersymmetrized SM with explicit soft breaking. There are, in fact, additional dimensionful couplings that one could write, such as

$$c_{ij}^u (h_d^\dagger \tilde{q}_i) \tilde{u}_j^c + c_{ij}^d (h_u^\dagger \tilde{q}_i) \tilde{d}_j^c + c_{ij}^e (h_u^\dagger \tilde{l}_i) \tilde{e}_j^c. \quad (19)$$

In the case of the MSSM, these terms do not reintroduce quadratic divergences either and are not generated by radiative corrections from $\mathcal{L}_{\text{soft}}$. Hence, it is consistent to

set them to zero. In an expansion in $1/M$, where M is the “messenger” scale, they are generated at higher orders.

Finally, let us remark that the soft masses $m_{h_u}^2, m_{h_d}^2$, and B_μ provide (for suitable values) for electroweak symmetry breaking, while the hierarchy $M_W \sim M_{\text{SUSY}} \ll M_X$, where M_X denotes one of the large scales $M_{\text{seesaw}}, M_{\text{GUT}}, M_{Pl}$, is stabilized by the softly broken supersymmetry. More fundamentally, the vacuum expectation values $\langle X \rangle$ may be due to dynamical supersymmetry breaking in the hidden sector, which can naturally generate the large hierarchy $M_{\text{SUSY}} \ll M_X$ by dimensional transmutation [24].

2.3 Patterns of SUSY breaking and their flavour

This section describes the expected flavour structure and mass spectrum in the two most popular mediation schemes. The first is gravity mediation, where the fact that SUSY is broken in the hidden sector is communicated to the MSSM particles by their gravitational interactions, which are always present [25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. These effects include “anomaly-mediated” contributions as a subset. The second scheme is gauge mediation, where there is an additional “messenger” sector containing (supersymmetrically) heavy particles charged under the SM gauge group and with direct couplings to the hidden sector SUSY-breaking field(s) X . Here the flavour structure is very non-generic, being flavour-blind at the messenger scale.

2.3.1 Gravity mediation

Coupling a supersymmetric theory to Einstein gravity implies invariance under local supersymmetry transformation and the presence of a spin-3/2 gravitino field, as well as certain auxiliary fields for the gravitational supermultiplet. Apart from that, the theory is specified by a Kähler potential K , a superpotential W , plus a gauge kinetic function f , as in the nongravitational case. Since the SUSY-mediating effects are themselves $1/M_{Pl}$ -suppressed, nonrenormalizable terms have to be kept in all three functions. Upon integrating out the supergravity auxiliary fields, couplings between the hidden- and visible-sector fields are generated. If K , W , and f are generic functions of ϕ_i/M_{Pl} , subject only to constraints from the gauge symmetry, substituting expectation values for the SUSY-breaking vevs $\langle X \rangle$ leads to non-universal SUSY-breaking scalar masses. For instance, in the case of a single hidden-sector F -term expectation value $\langle X \rangle = \langle F_X \rangle \theta^2$, there will be a contribution

$$\Delta V_{\text{soft}} = \phi_i \phi_j^* \frac{\partial^2 k}{\partial \phi_i \partial \phi_j^*} \Big|_{\phi=0}, \quad (20)$$

where

$$k = \frac{\partial^2 K}{\partial X \partial X^*} \Big|_{X=0}.$$

This contribution to scalar soft masses arises from quartic terms in the Kähler potential, which are $1/M_{Pl}^2$ -suppressed,

hence receive $\mathcal{O}(1)$ (relative) contributions from Planck-scale physics. There is no a-priori reason why these should have any particular structure. Likewise, the trilinear terms in $\mathcal{L}_{\text{soft}}$ can have a generic flavour structure. On the other hand, supergravity models do not appear to lead to sizable values for the terms in (19) (which arise at higher orders in $1/M_{\text{Pl}}$). In fact, although the mediation of SUSY breaking in supergravity might be viewed as due to “light” particles (the gravitational supermultiplet), all effects of supersymmetry breaking in the hidden sector on the MSSM sector can be accounted for by higher-dimensional operators of the form (13),(14),(15), with $M \rightarrow M_{\text{Pl}}$. (In general, the effective renormalizable visible-sector superpotential couplings will also differ from their counterparts in W .)

In summary, the generic pattern for gravity mediation is the softly broken MSSM in its full generality. More details and discussions of specific models can be found e.g. in the reviews [35,36].

2.3.2 mSUGRA

Specific assumptions on the Kähler function K and the breakdown of supersymmetry in the hidden sector (such as dilaton domination in string-theory compactifications) lead to non-generic forms. In the popular mSUGRA (or CMSSM) scenario, it is assumed that all running soft scalar masses unify at a certain scale (usually, M_{GUT}),

$$m_{\tilde{q}ij}^2 = m_{\tilde{u}ij}^2 = m_{\tilde{d}ij}^2 = m_{\tilde{l}ij}^2 = m_{\tilde{e}ij}^2 = m_0^2 \delta_{ij}, \\ m_{\tilde{h}_u}^2 = m_{\tilde{h}_d}^2 = m_0^2. \quad (21)$$

Moreover, the gaugino masses unify,

$$m_{\tilde{b}} = m_{\tilde{w}} = m_{\tilde{g}} = m_{1/2}, \quad (22)$$

while the trilinear couplings satisfy

$$T_{ij}^f = a_0 Y_{ij}^f; \quad f = u, d, e. \quad (23)$$

This leaves a predictive scenario with 4 parameters m_0 , a_0 , $m_{1/2}$, and B_μ (moreover there is a constraint on μ from the observed weak scale). We emphasize that the CMSSM should be considered as a limit which may receive important corrections. While in certain scenarios one may hope that the CMSSM captures some of the most important “flavourless” aspects of sparticle phenomenology, such as spectra, production rates, etc., it is less clear whether it is a good approximation for flavour physics, as there usually are large – although perhaps not $\mathcal{O}(1)$ – corrections to its minimal flavour structure from subdominant contributions to K in specific models.

2.3.3 Anomaly mediation

Supergravity involves a universal class of contributions to the soft-supersymmetry breaking terms whose form is independent of the details of the hidden-sector dependence of K , W and f , related to anomalous breaking of scale invariance. This results in contributions to gaugino masses,

scalar masses, and trilinear couplings which are fixed in terms of the RGE β functions and anomalous dimensions γ up to one parameter $m_{3/2}$ of order the gravitino mass [37,38],

$$m_{\tilde{g}_a} = -\frac{\beta_a}{2g_a^2} m_{3/2}, \quad (24)$$

$$T_{ij}^f = -\mu \frac{dY_{ij}^f}{d\mu} m_{3/2}, \quad f = u, d, c \quad (25)$$

$$m_{\tilde{m}ij}^2 = \frac{1}{2} \left(\beta_a \frac{\partial \gamma_{ij}^m}{\partial g_a} + \sum_{fkl} \mu \frac{dY_{kl}^f}{d\mu} \frac{\partial \gamma_{ij}^m}{\partial Y_{kl}^f} \right) |m_{3/2}|^2, \\ m = q, u, d, l, e \quad (26)$$

(with analogous contributions to the soft masses and the B_μ parameter for the Higgs scalars). Their form is RG invariant, and there exist schemes where their form holds true to all orders. It is evident from (24), (25), (26) that in a perturbative theory such as the MSSM the contributions to $m_{\tilde{g}_a}$ and T^f arise at one loop, while those to $m_{\tilde{m}ij}^2$ arise at two loops. I.e. all masses are of the same (loop) order and are loop-suppressed relative to the gravitino mass.

From a flavour-perspective, these soft terms are highly non-generic. In particular, for sfermions of the first two generations the contributions from Yukawa terms are small. This implies that the contributions to their trilinear scalar couplings T are negligible and those to their soft masses are degenerate.

2.3.4 Gauge mediation

In gauge mediation models [39,40,41,42], additional “messenger” particles with SM gauge interactions and further couplings to the fields that break supersymmetry are present. Typically, they receive large masses M from their couplings to the supersymmetry-breaking field(s), which acquire vacuum expectation values of the form

$$\langle X \rangle = S_X + \theta^2 F_X.$$

Below the scale M , the messengers can be integrated out. If all relevant couplings are perturbative, this gives rise to terms of the form (13), (14), (15) in a calculable manner.⁶ The structure of the soft breaking terms is [41]

$$m_a = \frac{g_a^2}{16\pi^2} \frac{F_X}{S_X}, \quad (27)$$

$$m_{\tilde{m}ij}^2 = 2 \frac{F_X^2}{S_X^2} \left(C_3 \left(\frac{\alpha_3}{4\pi} \right)^2 + C_2 \left(\frac{\alpha_2}{4\pi} \right)^2 + y_m^2 \left(\frac{\alpha_1}{4\pi} \right)^2 \right) \delta_{ij}, \quad (28)$$

where $C_3 = 4/3$ for color triplets (zero for singlets) and $C_2 = 3/4$ for weak doublets (zero for singlets). Trilinear couplings T_{ij}^f arise at two loops, hence are suppressed.

⁶ A more general but in general non-calculable parameterization which applies to a class of strongly-coupled gauged mediation models has been recently given in [43]

The flavour structure of gauge-mediated soft terms is completely universal, i.e. sfermions of identical SM gauge charges are degenerate to leading order. This fact strongly suppresses flavour-changing neutral currents. In particular, there are no contributions to any FCNC process from one-loop squark-gluino or squark-neutralino diagrams, as the flavour sfermions are mass eigenstates and the only contributions to FCNC still arise from the vertices shown in Fig. 1. Radiative corrections modify this simple pattern. The dominant (logarithmic) effects are accounted for by RG-evolving the soft terms from the messenger scale down to the mass scale F_X/S_X of the superpartners. We note that even in the case of gauge mediation, gravity-mediated contributions will be present, but their relative importance will be of

$$M/M_{\text{Pl}} \sim S_X/M_{\text{Pl}},$$

such that gravity-mediated contributions are strongly suppressed for a low messenger scale.

2.3.5 Radiative corrections to the soft terms

RGE effects in the MSSM modify both the spectrum and the flavour structure of the soft terms. As far as the flavour is concerned, no “dangerous” flavour structures are generated if they were not present at the messenger scale. In particular, for flavour-blind initial conditions as in gauge mediation or in msugra, the RG-evolved soft terms still have a minimally flavour-violating structure as defined in [10], and are closely aligned with the fermion mass matrices (with near degeneracy between the first two generations).

2.3.6 Minimal flavour violation

The principle of minimal flavour violation declares that the flavour symmetry (6) of the gauge interactions is only broken by the Yukawa couplings. That is, all soft terms are functions of the Yukawa matrices such that the Lagrangian is invariant when the Yukawas are treated as spurions of the symmetry [10]. This allows then to show, for instance, that the structure of minimal flavour violation is preserved under (non-gravitational) radiative corrections, and leads to strong correlations between different flavour-changing phenomena [8, 10].

Such a setup might occur due to flavour dynamics promoting the Yukawa matrices to dynamical fields which are frozen to their vacuum expectation values (we know of no concrete realization). In the MSSM it is realized for the case of pure gauge mediation, which provides universal soft masses and negligible trilinear couplings at the messenger scale.

In summary, general gravity mediation and gauge mediation form two extreme cases of complexity of flavour-violating soft terms, which range from completely generic in (general) gravity mediation to minimally flavour-violating in gauge mediation (with a messenger scale $M \ll M_{\text{Pl}}$).

2.4 General parameterizations

A general parameterization of the soft-breaking terms that is immediately useful in low-energy flavour phenomenology is given by the set of three sfermion mass matrices, including the effects from electroweak symmetry breaking, in the super-CKM basis. Together with the gaugino mass parameters and the dimension-two soft terms in the Higgs sector (two of which can be traded for the weak scale v and the parameter $\tan \beta$) they comprise the full information contained in $\mathcal{L}_{\text{soft}}$. Moreover, this parameterization is free from redundancies, i.e., all parameters are physical.⁷ Each of the three mass matrices is the sum of a supersymmetric and a soft-breaking part,

$$\mathcal{M}_{\tilde{f}}^2 = \mathcal{M}_{\tilde{f}}^{2,\text{sym}} + \mathcal{M}_{\tilde{f}}^{2,\text{break}}, \quad (29)$$

where $\mathcal{M}_{\tilde{f}}^{2,\text{sym}}$ has been given in (8) and

$$\begin{aligned} \mathcal{M}_{\tilde{u}}^{2,\text{break}} &= \begin{pmatrix} V_{\text{CKM}} \hat{m}_{\tilde{Q}}^2 V_{\text{CKM}}^\dagger & \frac{v}{\sqrt{2}} \sin \beta \hat{T}_U^\dagger \\ \frac{v}{\sqrt{2}} \sin \beta \hat{T}_U & \hat{m}_{\tilde{u}}^2 \end{pmatrix} \\ &\equiv \begin{pmatrix} \mathcal{M}_{\tilde{u}LL}^{2,\text{break}} & \mathcal{M}_{\tilde{u}LR}^{2,\text{break}} \\ (\mathcal{M}_{\tilde{u}LR}^{2,\text{break}})^\dagger & \mathcal{M}_{\tilde{u}RR}^{2,\text{break}} \end{pmatrix}, \end{aligned} \quad (30)$$

$$\begin{aligned} \mathcal{M}_{\tilde{d}}^{2,\text{break}} &= \begin{pmatrix} \hat{m}_{\tilde{Q}}^2 & \frac{v}{\sqrt{2}} \cos \beta \hat{T}_D^\dagger \\ \frac{v}{\sqrt{2}} \cos \beta \hat{T}_D & \hat{m}_{\tilde{d}}^2 \end{pmatrix} \\ &\equiv \begin{pmatrix} \mathcal{M}_{\tilde{d}LL}^{2,\text{break}} & \mathcal{M}_{\tilde{d}LR}^{2,\text{break}} \\ (\mathcal{M}_{\tilde{d}LR}^{2,\text{break}})^\dagger & \mathcal{M}_{\tilde{d}RR}^{2,\text{break}} \end{pmatrix}, \end{aligned} \quad (31)$$

$$\begin{aligned} \mathcal{M}_{\tilde{e}}^{2,\text{break}} &= \begin{pmatrix} \hat{m}_{\tilde{L}}^2 & \frac{v}{\sqrt{2}} \cos \beta \hat{T}_E^\dagger \\ \frac{v}{\sqrt{2}} \cos \beta \hat{T}_E & \hat{m}_{\tilde{e}}^2 \end{pmatrix} \\ &\equiv \begin{pmatrix} \mathcal{M}_{\tilde{e}LL}^{2,\text{break}} & \mathcal{M}_{\tilde{e}LR}^{2,\text{break}} \\ (\mathcal{M}_{\tilde{e}LR}^{2,\text{break}})^\dagger & \mathcal{M}_{\tilde{e}RR}^{2,\text{break}} \end{pmatrix}. \end{aligned} \quad (32)$$

The mass matrix for sneutrinos $\mathcal{M}_{\tilde{\nu}}^2$ is identical to $\mathcal{M}_{\tilde{e}LL}^{2,\text{break}} = m_{\tilde{L}}^2$ in the flavour basis (up to flavour-conserving D -terms, and neglecting infinitesimal supersymmetric contributions from neutrino masses). The hatted mass matrices are identified with the quadratic terms in $\mathcal{L}_{\text{soft}}$ in the super-CKM basis as $\hat{m}_{\tilde{Q}}^2 = m_{\tilde{Q}}^2$, $\hat{m}_{\tilde{u}}^2 = (m_{\tilde{u}}^2)^T$, $\hat{m}_{\tilde{d}}^2 = (m_{\tilde{d}}^2)^T$, $\hat{m}_{\tilde{L}}^2 = m_{\tilde{L}}^2$, $\hat{m}_{\tilde{e}}^2 = (m_{\tilde{e}}^2)^T$.

Several remarks are in order.

1. The LR masses are proportional to the electroweak scale. Hence they are suppressed by v/M_{SUSY} in the limit of a large SUSY-breaking scale.

⁷ after removing a freedom in phases of fields by choosing $m_{\tilde{g}} > 0$, $B_\mu > 0$ real.

2. We recall that the neutral fermion-sfermion-gaugino and fermion-sfermion-higgsino couplings are flavour-diagonal in the super-CKM basis, while the charged couplings are governed by the CKM matrix. However, unless assumptions about the mechanism of SUSY breaking are made, the matrices entering (30)–(32) are in general nondiagonal, subject only to certain hermiticity conditions.
3. The left-left sectors of the up- and down-type squark mass matrices are not independent. In particular, at least one of them will violate flavour as soon as either deviates from a multiple of the unit matrix. Such deviations are already induced by leading-logarithmic corrections proportional to y_t^2 , which induce corrections that are aligned with up-type quarks. Hence even in gauge mediation models, there will be FCNC induced by gluino-squark loops [10].

The nonminimal flavour violation is conveniently parameterized in terms of the off-diagonal mass matrix elements, or equivalently the “ δ ” parameters,

$$(\delta_{ij}^{\tilde{f}})_{AB} \equiv \frac{[\mathcal{M}_{\tilde{f}AB}^{2,\text{break}}]_{ij}}{M^2}, \quad (33)$$

where $A, B = L, R$ and M is a mass scale of order the sfermion mass eigenvalues. (A popular flavour-dependent choice is to use $M^2 = \sqrt{[(\mathcal{M}_f^2)_{XX}]_{ii}[(\mathcal{M}^2)_{YY}]_{jj}}$ in $(\delta_{ij}^{\tilde{f}})_{AB}$.) When the δ parameters are small, it is possible to treat them as perturbations [12]. This is in most cases justified by the constraints on them calculated in such an expansion; however, caution must be taken to expand to sufficiently high orders to account for all leading effects.

From the large number of flavour-violating parameters, it is evident that the MSSM generally entails deviations from the SM predictions for flavour-violating processes. This opens the possibility to either observe supersymmetry indirectly or to constrain its parameters (flavour-violating as well as flavour-conserving ones). One virtue of flavour-violating processes is the large number of observables and the availability of theoretical tools for rather precise predictions for many of them. On the other hand, for generic $\delta \sim 1$ and TeV-scale sparticle masses (such as in generic gravity-mediated setups) experimental bounds on FCNC processes such as $B \rightarrow X_s \gamma$ are violated (the “SUSY flavour problem”). However, as elaborated above, the structure of $\mathcal{L}_{\text{soft}}$ is intimately tied to the unknown mechanism of SUSY breaking. For instance, gauge-mediation models have little trouble in satisfying the low-energy constraints from flavour physics because the SUSY breaking is transferred by flavour-blind gauge interactions at relatively low scales.

2.5 Constraints from vacuum stability

Before discussing the consequences of non-minimal flavour violation for phenomenology (and vice versa), we note that, independently of experimental searches for FCNC

Table 3. CCB and UFB constraints on flavour-violating parameters $(\delta_{ij}^{\tilde{f}})_{LR}$ [44]

	u	d	e
12	3.1×10^{-2}	2.9×10^{-4}	3.6×10^{-4}
13		10^{-2}	6.1×10^{-3}
23		6.1×10^{-3}	6.1×10^{-3}

or other low-energy SUSY-induced effects, there exists a class of “theoretical” constraints on the soft terms following from the requirement of the correct electroweak symmetry breaking pattern, which also constrain flavour-off-diagonal mass matrix elements. The reason is that, with many scalar fields in the MSSM, there are flat directions in field space (where the supersymmetric contribution to the scalar potential vanishes). Unless the trilinear couplings satisfy certain constraints, the softly broken scalar potential has charge- or colour-breaking (CCB) minima, or is unbounded from below (UFB) in certain directions. The authors of [44] obtained CCB and UFB bounds on the $(\delta_{ij}^{\tilde{f}})_{LR}$, which respectively read:

$$(\delta_{ij}^{\tilde{u}})_{LR} < m_{u_k} \frac{\sqrt{m_{u_{Li}}^2 + m_{u_{Rj}}^2 + m_{h_u}^2}}{M^2}, \quad (34)$$

$$(\delta_{ij}^{\tilde{d}})_{LR} < m_{d_k} \frac{\sqrt{m_{d_{Li}}^2 + m_{d_{Rj}}^2 + m_{h_d}^2}}{M^2}, \quad (35)$$

$$(\delta_{ij}^{\tilde{e}})_{LR} < m_{e_k} \frac{\sqrt{m_{e_{Li}}^2 + m_{e_{Rj}}^2 + m_{h_d}^2}}{M^2}, \quad (36)$$

and

$$(\delta_{ij}^{\tilde{u}})_{LR} < m_{u_k} \frac{\sqrt{m_{u_{Li}}^2 + m_{u_{Rj}}^2 + m_{e_{Li}}^2 + m_{e_{Rj}}^2}}{M^2}, \quad (37)$$

$$(\delta_{ij}^{\tilde{d}})_{LR} < m_{d_k} \frac{\sqrt{m_{d_{Li}}^2 + m_{d_{Rj}}^2 + \frac{1}{2}(m_{e_{Li}}^2 + m_{e_{Rj}}^2)}}{M^2}, \quad (38)$$

$$(\delta_{ij}^{\tilde{e}})_{LR} < m_{e_k} \frac{\sqrt{3(m_{e_{Li}}^2 + m_{e_{Rj}}^2)}}{M^2}, \quad (39)$$

where always $k = \max(i, j)$. Not only are these bounds in several cases stronger than those from experimental data, but an important virtue is that they do not become weaker as the SUSY scale is raised; this is different from the FCNC bounds reviewed below. (The equivalent bounds on the trilinear T -parameters themselves scale like yM , where y is the larger of the Yukawa couplings involved.) Numerical values are listed in Table 3, corresponding to universal sfermion masses.

3 Low-energy phenomena

Each of the flavour-violating parameters $(\delta_{ij}^{\tilde{f}})_{AB}$ changes flavour by one unit. Hence generically, they mediate flavour-changing weak decays of B , D , K mesons as well as charged

leptons, such as $B \rightarrow X_s \gamma$ or $\tau \rightarrow \mu \gamma$, at linear order and $\Delta F = 2$ processes, namely particle-antiparticle mixing of neutral mesons, at quadratic order. Such processes provide, at present, stringent constraints on supersymmetric parameters, and eventually discovery and “measurement” potential for supersymmetry.

Most observables receive contributions from both the Standard Model and from supersymmetry. Schematically,

$$\mathcal{A} = \mathcal{A}_{\text{SM}} + \mathcal{A}_{\text{SUSY}}. \quad (40)$$

A decay rate provides a (schematic) constraint

$$|\mathcal{A}_{\text{SUSY}}|^2 + 2 \text{Re} \mathcal{A}_{\text{SUSY}}^* \mathcal{A}_{\text{SM}} \quad (41)$$

$$= \Gamma_{\text{exp}}(1 \pm \Delta^{(\text{exp})}) - |\mathcal{A}_{\text{SM}}|^2(1 \pm \Delta^{(\text{SM})}). \quad (42)$$

By this we mean that the left-hand side, which can be calculated in principle from the parameters of the SM and the MSSM, is constrained to lie within a range (right-hand side) determined by the experimental range (or bound) on the decay rate and uncertainties on the theoretical prediction of its pure SM contribution. In the case of meson-antimeson mixing in principle information on the complex $\mathcal{A}_M \equiv \mathcal{A}(\bar{M} \rightarrow M)$ is accessible (once phase conventions are fixed), i.e.

$$\begin{aligned} \mathcal{A}_{M,\text{SUSY}} &\equiv (C_M e^{i2\phi_M} - 1) \mathcal{A}_{\text{SM}} \\ &= \mathcal{A}_{M,\text{exp}}(1 \pm \Delta^{(\text{exp})}) - \mathcal{A}_{M,\text{SM}}(1 \pm \Delta^{(\text{SM})}) \end{aligned} \quad (43)$$

(here strictly speaking one has two independent equations, including errors, for magnitude and complex phase). The right-hand sides often involve a “cancellation”: the flavour observables measured so far are consistent with no SUSY contributions. Hence it is important to improve the precision on $\Delta^{(\text{exp})}$ and $\Delta^{(\text{SM})}$ as much as possible; depending on the mode this is mainly a theory or an experimental challenge.

Conversely, for the SUSY contributions on the left-hand side the leading dependence on SUSY parameters is enough.

3.1 Theoretical framework

Weak decays are often either loop-induced in the Standard Model or receive important loop corrections. Moreover, they are multi-scale problems, involving large logarithms of physical scales $M_{\text{SUSY}} \gg M_L$, where M_L denotes a generic mass of the initial- and final-state particles involved (B , D , τ , μ , light mesons, leptons and neutrinos). In view of the smallness of the latter compared to the weak and SUSY scales, effective theories provide a framework for separating (factorizing) the contributions from the weak scale and above from low-energy QCD and QED effects. Apart from the huge simplification in theoretical expressions for the observables, effective theories are the appropriate tool to curb large logarithms $\ln M_W/m_L$ or $\ln M_{\text{SUSY}}/m_L$, and to separate nonperturbative QCD effects into a limited number of well-defined hadronic matrix elements.

3.1.1 SUSY scale and decoupling

Consider first the case $M_{\text{SUSY}} \gg M_W$. In this case, integrating out the superpartners one readily obtains a set of higher-dimensional local operators built from SM fields (including the Higgs and weak bosons), suppressed by powers of M_{SUSY} . As is well known, the unique operator at dimension 5 violates lepton number by two units and is not generated from the MSSM. A complete set of operators up at dimension six has been given in [45]. Without computing any loops, this implies that SUSY contributions (or those from any other new physics with a new large mass scale M) to decay amplitudes decouple as

$$\Delta\{BR, A_{\text{CP}}, \dots\}_{\text{SUSY}} \sim \frac{\delta_{ij}^{\tilde{f}}}{M^2}, \quad (44)$$

while effects in meson-antimeson mixing decouple like

$$\Delta\{\epsilon_K, \Delta M_{B,D,K}, S_{J/\psi K}, S_{J/\psi \phi}, \dots\}_{\text{SUSY}} \sim \frac{(\delta_{ij}^{\tilde{f}})^2}{M^2}, \quad (45)$$

in agreement with the decoupling theorem. In particular, the flavour-changing vertices of the Z and the magnetic vertices of the photon and the gluon decouple as M^{-2} , despite being apparently of dimension 4 and 5, respectively. Conversely, it means that experimental bounds on the δ parameters scale, in general, linearly with M for mixing and quadratically for decays. Note the difference to the vacuum-stability constraints (34)–(39).

In general, of course, one expects that at least some of the superpartners are close to the weak scale on naturalness grounds, such that in practice it is often necessary to integrate out sparticles and the weak scale at the same time.

3.1.2 Weak scale

The appropriate tool to separate the heavy scales from the low-energy QCD effects and curb large QCD (and QED) logarithms is the effective weak Hamiltonian (see [46] for a review of the formalism). Integrating out the weak scale, one has

$$\mathcal{A}(i \rightarrow f) = \langle f | \mathcal{H}_{\text{eff}} | i \rangle, \quad (46)$$

where the weak Hamiltonian

$$\mathcal{H}_{\text{eff}} = \sum_k C_k(\mu) Q_k(\mu) \quad (47)$$

is given in terms of local operators Q_k constructed from the “light” SM fields ($u, d, s, c, b, \tau, \mu, e$, photon, gluons, and neutrinos) and of Wilson coefficients C_k encapsulating the effects of the weak interactions and the sparticles, and μ is a renormalization (factorization) scale. Radiative corrections from above the scale μ are contained in the $C_k(\mu)$.

The Wilson coefficients are evolved to lower scales according to RG equations

$$\mu \frac{d}{d\mu} C_k(\mu) = \gamma_{lk}(\alpha_s(\mu), \alpha) C_l(\mu), \quad (48)$$

where (γ_{lk}) are perturbative, process-independent anomalous-dimension matrices.

3.1.3 Lower scales

In a purely leptonic decay such as $\tau \rightarrow \mu \gamma$, the matrix element of the weak hamiltonian can be simply calculated in perturbation theory. (In fact, in this case the use of the weak Hamiltonian is not very essential due to the absence of large radiative corrections.) For the large amount of data that involve hadrons, one has only

$$\mathcal{A}(i \rightarrow f) = \sum_k C_k(\mu) \langle f | Q_k(\mu) | i \rangle \equiv \sum_k C_k(\mu) B_k(i, f), \quad (49)$$

where μ is optimally chosen of order of the mass of i . The hadronic matrix elements $\langle f | Q_k(\mu) | i \rangle$ are usually nonperturbative and only calculable in some cases. The latter include matrix elements for meson-antimeson mixing, which can be obtained using numerical lattice QCD methods. Other methods include QCD sum rules based on the operator product expansions (for inclusive and some exclusive B , as well as hadronic τ decays) and collinear expansions (for some exclusive B decays), chiral perturbation theory in K decays, and the use of approximate flavour symmetries of QCD to reduce the number of independent hadronic matrix elements; all of these have systematics controlling which is a theoretical challenge.

3.2 $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, $B_s - \bar{B}_s$, and $D^0 - \bar{D}^0$ mixing

Meson mixings are $\Delta F = 2$ processes. At one loop, the effective $\Delta F = 2$ hamiltonian to meson-antimeson oscillations is solely due to box diagrams. Complete operator bases have been given in [1, 47]. For $\Delta B = \Delta S = 2$ transitions ($B_s - \bar{B}_s$ mixing), one choice consists of the five operators

$$Q_1 = (\bar{s}_L^a \gamma_\mu b_L^a)(\bar{s}_L^b \gamma^\mu b_L^b), \quad (50)$$

$$Q_2 = (\bar{s}_R^a b_L^a)(\bar{s}_R^b b_L^b), \quad (51)$$

$$Q_3 = (\bar{s}_R^a b_L^b)(\bar{s}_R^b b_L^a), \quad (52)$$

$$Q_4 = (\bar{s}_R^a b_L^a)(\bar{s}_L^b b_R^b), \quad (53)$$

$$Q_5 = (\bar{s}_R^a b_L^b)(\bar{s}_L^b b_R^a) \quad (54)$$

(a, b colour indices), plus operators $\tilde{Q}_{1,2,3}$ obtained by flipping the chiralities of all fermions in $\tilde{Q}_{1,2,3}$. The operator basis for $B_d - \bar{B}_d$, $D^0 - \bar{D}^0$, and $K^0 - \bar{K}^0$ mixing are identical up to obvious substitutions of quark flavours (in the case of $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixing, there are also sizable “long-distance” contributions which cannot be written in terms of local four-quark operators at the weak scale).

Only Q_1 is generated in the SM (to excellent approximation), following from $W - t$ boxes (Fig. 2.) This results in

$$C_1^{\text{SM}} = \frac{G_F^2 M_W^2}{16\pi^2} (V_{tb} V_{ts}^*)^2 4 S(x_t), \quad (55)$$

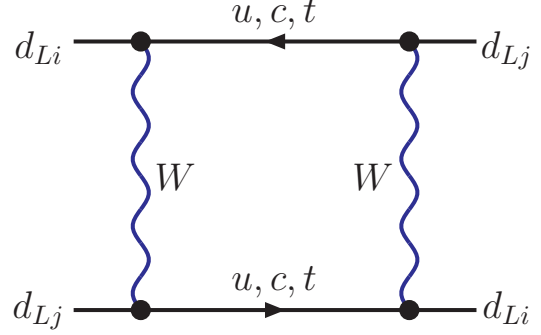


Fig. 2. SM diagram for neutral meson-antimeson mixing. (Diagrams including Goldstone bosons in R_ξ gauge not shown.)

where S [48] is listed in appendix A. SM NLO QCD corrections are reviewed in [46].

Supersymmetric contributions have been computed in [1, 49, 50, 51, 52, 53, 54, 55, 56, 57]. Since each δ changes flavour by one unit, the leading contributions are of second order in these parameters. The simplest way to obtain the second-order terms is to work in the “mass-insertion approximation”, where the off-diagonal sfermion-mass-matrix elements are treated as perturbations (Fig. 3). For instance, for two LL mass insertions, diagram 3 (a) (to zeroth order in external momenta, and neglecting mass differences between the squarks in the loop) is proportional to

$$\begin{aligned} & \int d^4 k \frac{k^2 (\mathcal{M}_{\tilde{d}LL}^2)_{sb}^2}{(k^2 - m_{\tilde{g}}^2)^2 (k^2 - m_{\tilde{q}}^2)^4} \\ &= \frac{(\delta_{sb}^{\tilde{d}})_{LL}^2 (m_{\tilde{q}}^2)^2 d^2}{6 (dm_{\tilde{q}}^2)^2} \int d^4 k \frac{k^2}{(k^2 - m_{\tilde{g}}^2)^2 (k^2 - m_{\tilde{q}}^2)^2}. \end{aligned} \quad (56)$$

The full result for the gluino-squark contributions reads [1]

$$C_1 = -\epsilon [24x f_6(x) + 66 \tilde{f}_6(x)] (\delta_{sb}^{\tilde{d}})_{LL}^2, \quad (57)$$

$$\tilde{C}_1 = -\epsilon [24x f_6(x) + 66 \tilde{f}_6(x)] (\delta_{sb}^{\tilde{d}})_{RR}^2, \quad (58)$$

$$C_2 = -\epsilon 204x f_6(x) (\delta_{sb}^{\tilde{d}})_{RL}^2, \quad (59)$$

$$\tilde{C}_2 = -\epsilon 204x f_6(x) (\delta_{sb}^{\tilde{d}})_{LR}^2, \quad (60)$$

$$C_3 = \epsilon 36x f_6(x) (\delta_{sb}^{\tilde{d}})_{RL}^2, \quad (61)$$

$$\tilde{C}_3 = \epsilon 36x f_6(x) (\delta_{sb}^{\tilde{d}})_{LR}^2, \quad (62)$$

$$\begin{aligned} C_4 = & -\epsilon [504x f_6(x) - 72 \tilde{f}_6(x)] (\delta_{sb}^{\tilde{d}})_{LL} (\delta_{sb}^{\tilde{d}})_{RR} \\ & + \epsilon 132 \tilde{f}_6(x) (\delta_{sb}^{\tilde{d}})_{LR} (\delta_{sb}^{\tilde{d}})_{RL}, \end{aligned} \quad (63)$$

$$\begin{aligned} C_5 = & -\epsilon [24x f_6(x) + 120 \tilde{f}_6(x)] (\delta_{sb}^{\tilde{d}})_{LL} (\delta_{sb}^{\tilde{d}})_{RR} \\ & + \epsilon 180 \tilde{f}_6(x) (\delta_{sb}^{\tilde{d}})_{LR} (\delta_{sb}^{\tilde{d}})_{RL}. \end{aligned} \quad (64)$$

Here $(\delta_{ij}^{\tilde{d}})_{RL} \equiv (\delta_{ji}^{\tilde{d}})_{LR}^*$, $\epsilon = \alpha_s^2 / (216 m_{\tilde{q}}^2)$, $x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2$, and $f_6(x)$, $\tilde{f}_6(x)$ are dimensionless loop functions (appendix A)

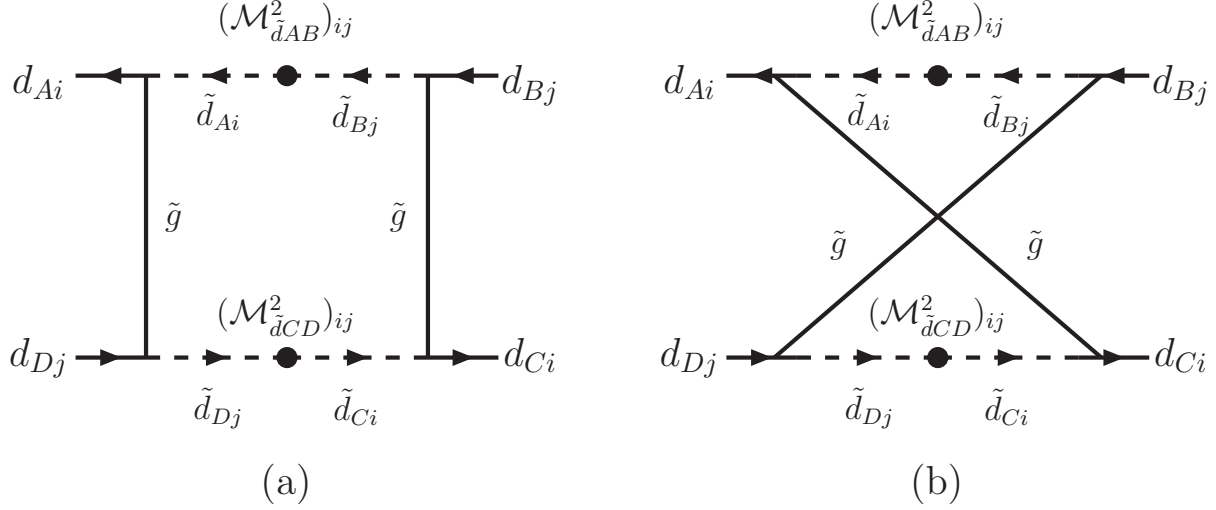


Fig. 3. Diagrams for meson-antimeson mixing. A, B, C, D denote chiralities of the quarks (and squarks). The blobs are flavour-changing “mass insertions”.

There are also chargino-up-squark contributions. These can be competitive with the gluino-squark contributions if the charginos are lighter than the gluinos, as tends to be the case in GUT scenarios. There are always “minimally flavour-violating” contributions, which are proportional to the same CKM factors as the SM contributions. Of interest here are the additional contributions due to nonvanishing δ^u parameters. Neglecting terms suppressed by small CKM elements or small Yukawa couplings, only C_1 receives a contribution [58]

$$C_1 = -\frac{G_F \alpha}{\sqrt{2} \pi \sin^2 \theta_W} \frac{M_W^2}{m_{\tilde{q}}^2} \times \frac{1}{20} \left[([\delta_{ij}^u]_{LL})^2 - \frac{2}{3} (\delta_{ij}^u)_{LL} (\delta_{it}^u)_{LR} (\delta_{jt}^u)_{LR}^* + \frac{1}{7} [(\delta_{it}^u)_{LR} (\delta_{jt}^u)_{LR}^*]^2 \right]. \quad (65)$$

Note that the chargino contributions involve either a LL mass insertion or a double LR one on each squark line; for the latter, only those involving a stop can be relevant according to Table 3. (For $B - \bar{B}$ mixing, there may be additional operators [59].)

If $\tan \beta$ is large, there are in principle also terms proportional to y_b that could be important. In that case, however, Higgs double-penguin diagrams are often dominant and require a modified treatment [60, 61, 62, 63].

3.2.1 $K - \bar{K}$ mixing and constraints on δ 's

$K - \bar{K}$ oscillations proved their discovery potential in estimating the charm quark mass before its observation [64], as well as in the discovery of (indirect) CP violation

[65], later giving information on the CP-violating phase in the CKM matrix. The possibility of large SUSY contribution was recognized early on [66, 67, 68, 69, 70], and ΔM_K and ϵ_K still provide the strongest FCNC constraints on the MSSM parameters. The mass difference ΔM_K and the CP-violating parameter ϵ_K follow from the effective $\Delta F = 2$ Hamiltonian,

$$\Delta M_K \propto 2 \sum_i B_i \operatorname{Re} C_i, \quad (66)$$

$$\epsilon_K \propto \frac{e^{i\pi/4}}{\sqrt{2} \Delta M_K} \sum_i B_i \operatorname{Im} C_i, \quad (67)$$

where $B_i \equiv \langle K | Q_i | \bar{K} \rangle$. The hadronic matrix elements B_i contain low-energy QCD effects and require nonperturbative methods such as (numerical) lattice QCD, see e.g. [71, 72, 73].⁸ Moreover, ΔM_K is afflicted by long-distance contributions which are believed to be not much larger than the SM short-distance contribution but are difficult to estimate. Nevertheless, in view of the strong CKM suppression of the SM contribution, even a rough estimate of the B_i translates into strong constraints on $s \rightarrow d$ flavour violation parameters. The procedure is as follows [1]:

- Write out the expression for the observable (here, ϵ_K or ΔM_K) as linear combination of (products of) δ -parameters, inserting estimates of the hadronic matrix elements.
- Require that each term at most saturates the experimental result.

⁸ Usually, the hadronic matrix elements are normalized to their values obtained from PCAC in “vacuum-insertion approximation”. This normalization is included in the B_i here.

Table 4. Constraints from $K - \bar{K}$ mixing on δ parameters [1, 2] for $m_{\tilde{q}} = 500$ GeV and $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2 = 1$. Bounds vary by less than one order of magnitude in the range $x \in [0.3, 4.0]$. For the bounds on LR mass insertions, $(\delta_{ds}^{\tilde{d}})_{LR} \sim (\delta_{ds}^{\tilde{d}})_{RL}$ has been assumed [1].

Quantity	upper bound
$\sqrt{ \text{Re}(\delta_{ds}^{\tilde{d}})_{LL}^2 }$	4.0×10^{-2}
$\sqrt{ \text{Re}(\delta_{ds}^{\tilde{d}})_{RR}^2 }$	4.0×10^{-2}
$\sqrt{ \text{Re}(\delta_{ds}^{\tilde{d}})_{LR}^2 }$	4.4×10^{-3}
$\sqrt{ \text{Re}(\delta_{ds}^{\tilde{d}})_{LL}(\delta_{ds}^{\tilde{d}})_{RR} }$	2.8×10^{-3}
$\sqrt{ \text{Im}(\delta_{ds}^{\tilde{d}})_{LL}^2 }$	3.2×10^{-3}
$\sqrt{ \text{Im}(\delta_{ds}^{\tilde{d}})_{RR}^2 }$	3.2×10^{-3}
$\sqrt{ \text{Im}(\delta_{ds}^{\tilde{d}})_{LR}^2 }$	3.5×10^{-4}
$\sqrt{ \text{Im}(\delta_{ds}^{\tilde{d}})_{LL}(\delta_{ds}^{\tilde{d}})_{RR} }$	2.2×10^{-4}

This is clearly a crude procedure. Typically, there will be some degree of constructive or destructive interference between different terms and with the SM contribution. However, in view of the large number of parameters this is a reasonable approach: Even with a more precise determination of the Standard Model contribution (including the relevant hadronic matrix elements and nonlocal contributions), the possible presence of cancellations prohibits a significant improvement of this type of constraint without further assumptions.

For the case of $K - \bar{K}$ mixing, this leads to the bounds [1, 2] shown in Table 4 (assuming gluino-squark dominance and absence of cancellations). This is one case of the ‘‘SUSY flavour problem’’. The constraints become weaker as the SUSY scale is increased, scaling like M , as is required by the decoupling properties discussed above and is evident from (57)–(64). At any rate, this ‘‘problem’’ looks less severe when considering that the corresponding CKM factor $V_{td}^* V_{ts} = \mathcal{O}(10^{-4})$ is also much smaller than its ‘generic’ value $\mathcal{O}(1)$, and that the LR δ parameters are $\mathcal{O}(v/M)$; moreover, already the vacuum-stability bounds considered in Subsection 2.5 show that the squark mass matrices (in particular, the left-right elements deriving from the T -parameters) must be far from generic. As previously noted, the problem is completely removed, for instance, in gauge mediation, which forms a special case of the framework of minimal flavour violation [7, 8, 9, 10].

Slightly more stringent constraints on the mass insertions have been obtained in [74], which uses better hadronic matrix elements from a lattice calculation and imposes the (stronger) requirement that for Δm_K and ϵ_K , the SUSY contribution from any single biproduct of mass insertions does not exceed the difference between the (short-distance) SM contribution and the measured value.

Table 5. Constraints from $D^0 - \bar{D}^0$ mixing on δ^u parameters. Here we have rescaled the results from [1] (for $m_{\tilde{q}} = 500$ GeV and $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2 = 1$) to account for the replacement of the old experimental upper bound $\Delta M_D < 1.32 \times 10^{-10}$ MeV used there by the current one-sigma upper bound $\Delta M_D < 1.99 \times 10^{-10}$ MeV [82], otherwise the remarks concerning Table 4 apply.

Quantity	upper bound
$\sqrt{ \text{Re}(\delta_{uc}^{\tilde{u}})_{LL}^2 }$	3.9×10^{-2}
$\sqrt{ \text{Re}(\delta_{uc}^{\tilde{u}})_{RR}^2 }$	3.9×10^{-2}
$\sqrt{ \text{Re}(\delta_{uc}^{\tilde{u}})_{LR}^2 }$	1.20×10^{-2}
$\sqrt{ \text{Re}(\delta_{uc}^{\tilde{u}})_{LL}(\delta_{uc}^{\tilde{u}})_{RR} }$	6.6×10^{-3}

One has

$$|(\delta_{ds}^{\tilde{d}})_{LL}|, |(\delta_{ds}^{\tilde{d}})_{RR}| < \mathcal{O}(10^{-2}), \quad (68)$$

$$|(\delta_{ds}^{\tilde{d}})_{LR}|, |(\delta_{ds}^{\tilde{d}})_{RL}| < \mathcal{O}(10^{-3}), \quad (69)$$

$$|(\delta_{ds}^{\tilde{d}})_{LL} \cdot (\delta_{ds}^{\tilde{d}})_{RR}| < \mathcal{O}(10^{-7}) \quad (70)$$

for $M \sim m_{\tilde{q}} \sim m_{\tilde{g}} \sim 500$ GeV [74]. In particular, the bounds on products of LL and RR insertions are strengthened. In that paper, specifics on the procedure and more detailed numerical results can be found.

3.2.2 $D - \bar{D}$ mixing

With the observation of $D - \bar{D}$ oscillations in 2007 at the B -factories [75, 76, 77], particle-antiparticle mixing has now been established in all four of the neutral meson-antimeson systems. Unfortunately it is very difficult to quantify the SM contribution to the mixing amplitude, as it is completely long-distance dominated and rather uncertain. Hence possible new-physics short-distance contributions are only limited by the experimental upper bound (barring cancellations with the unknown SM contribution), which has now been replaced and improved by a concrete measurement. The situation is very different for possible CP violation: as the mixing amplitude must have a negligible weak phase in the SM (being due to only the first two generations), mixing-induced CP violation in D decays would clearly signal non-SM physics. Implications of the improved bound for new physics, including supersymmetry, have been discussed in [78, 79, 80, 81]. In particular, since $(\delta_{12}^u)_{LL}$ and $(\delta_{12}^d)_{LL}$ mass insertions are related to each other and the mass splitting between \tilde{d}_L and \tilde{s}_L by the Cabibbo angle, it appears less likely that squark and quark masses are merely aligned; rather a near degeneracy of at least the left-chiral squarks of the first two generations is necessary [78, 79, 80].

Considering gluino-squark box diagrams and following the same procedure as outlined for $K^0 - \bar{K}^0$ mixing above, the constraints on the up-squark mass matrices listed in Table 5 result.

3.2.3 $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing

Here the mixing amplitudes

$$\mathcal{A}(\bar{B}_q \rightarrow B_q) \propto M_{12}^q - \frac{i}{2} \Gamma_{12}^q \quad (71)$$

($q = d, s$) are completely short-distance dominated. Hence the theoretical expression

$$\Delta M_{B_q} \propto |M_{12}^q| \sim f_{B_q}^2 \left| \sum B_i R_i C_i \right|, \quad (72)$$

where f_{B_q} denotes a decay constant and R_i a known (up to mild uncertainties) factor defined by the vacuum insertion approximation ($\langle B_q | Q_i | \bar{B}_q \rangle \equiv f_{B_q} R_i B_i$) can be directly compared to the experimental values [83, 84]

$$\Delta M_{B_q} = (0.507 \pm 0.005) \text{ ps}^{-1}, \quad (73)$$

$$\Delta M_{B_s} = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1}. \quad (74)$$

In both cases, the theory error is fully dominated by f_{B_q} . For instance, $\Delta M_{B_s}^{\text{SM}} \approx (16 \dots 27) \text{ ps}^{-1}$ is achievable depending on which value of f_{B_s} [85, 86, 87, 88] is used. Conversely, the residual “bag factors” B_i have much smaller uncertainties. The SM prediction is consistent with the experimental value. However, the theoretical range clearly allows for several tens of percent of new-physics contribution, in particular if it is mostly imaginary (i.e. CP violating). See also Section 5 below. Combined with the fact that the remaining (perturbative, non-CKM) uncertainties are at the 1–2 percent level, this underlines the importance of the ongoing efforts to obtain these nonperturbative parameters on the lattice with a high precision.

On the other hand, the weak phase $\phi_d = \arg M_{12}^d$ governs mixing-decay interference, and can be extracted cleanly from the time-dependent CP asymmetry in $B \rightarrow J/\psi K_S$ decay (with theoretical uncertainties of order 1–2 %), giving [83]

$$\sin \phi_d = 0.680 \pm 0.025. \quad (75)$$

In the SM, $\phi_d = 2\beta$, but this does not hold in the presence of new flavour violation. Further information on the mixing phase (more precisely, on $\arg(-M_{12}/\Gamma_{12})$, which in the MSSM and other new-physics scenarios without new light degrees of freedom can be related to $\arg M_{12}$ using a robust theoretical framework, see [89] for a recent theoretical account) can be obtained from flavour-specific B_d and B_s decays, notably the so-called semileptonic CP asymmetries.

One of the most exciting developments in the past year has been the first measurement of the mixing phase ϕ_s^9 in

⁹ We caution the reader that the literature abounds with conventions and meanings of the symbol ϕ_s and similar phases used in parameterizing new-physics effects. Here, $\phi_s \equiv \arg M_{12}^s$ in the standard CKM parameterization [Ref. [89] uses ϕ_s to denote the (CKM-parameterization-invariant) phase $\arg(-M_{12}^s/\Gamma_{12}^s)$]. Furthermore, in the standard parameterization one has $\beta \approx -\arg V_{td}$, however $\beta_s \approx +\arg(-V_{ts})$.

Table 6. Constraints from $B_d^0 - \bar{B}_d^0$ [1] and $B_s^0 - \bar{B}_s^0$ mixing on δ^d parameters. For the latter, we have rescaled the B_d^0 bounds to account for the larger mass difference $\Delta M_{B_s} = (117.0 \pm 0.8) \times 10^{-10} \text{ MeV}$ (as opposed to $\Delta M_{B_d} = (3.337 \pm 0.033) \times 10^{-10} \text{ MeV}$) [82] and the non-unit ratio of the decay constants, which we take to be $f_{B_s}/f_{B_d} = 1.2$ in agreement with lattice-QCD results (see [88] for a review). Otherwise the remarks concerning Table 4 apply.

Quantity	upper bound
$\sqrt{ \text{Re}(\delta_{db}^d)_{LL}^2 }$	9.8×10^{-2}
$\sqrt{ \text{Re}(\delta_{db}^d)_{RR}^2 }$	9.8×10^{-2}
$\sqrt{ \text{Re}(\delta_{db}^d)_{LR}^2 }$	3.3×10^{-2}
$\sqrt{ \text{Re}(\delta_{db}^d)_{LL}(\delta_{db}^d)_{RR} }$	1.8×10^{-2}
$\sqrt{ \text{Re}(\delta_{sb}^d)_{LL}^2 }$	4.8×10^{-1}
$\sqrt{ \text{Re}(\delta_{sb}^d)_{RR}^2 }$	4.8×10^{-1}
$\sqrt{ \text{Re}(\delta_{sb}^d)_{LR}^2 }$	1.62×10^{-2}
$\sqrt{ \text{Re}(\delta_{sb}^d)_{LL}(\delta_{sb}^d)_{RR} }$	8.9×10^{-2}

the B_s system by the D0 and CDF collaborations at Fermilab from an analogous time-dependent CP asymmetry in $B_s \rightarrow J/\psi \phi$ decays. The current 68 % CL average of CDF and D0 data reads [90]

$$-65^\circ < \phi_s < -27^\circ \vee -152^\circ < \phi_s < -113^\circ. \quad (76)$$

The statistical accuracy will be further reduced by CDF and D0 and by LHCb, with a goal of about 1° by 2013 [91]. As in the SM $\phi_s \approx -2\beta_s = -2.2^\circ \pm 0.6^\circ$ [89], such a mixing-induced asymmetry if confirmed would be an unambiguous, theoretically clean signal of not only new but nonminimally flavoured physics (see Section 5 below).

Several authors have considered the impact of the oscillation measurements on the general MSSM. Constraints on δ 's analogous to those from the neutral K and D systems follow from $B_d - \bar{B}_d$ (see [1, 92]) and from $B_s - \bar{B}_s$ (see [93], which treats the large $\tan \beta$ case where also Higgs double-penguin diagrams can make important contributions, and [94, 95]) mixing data. Bounds following from the mass differences according to the prescription of [1] are listed in Table 6. See also Subsection 3.5, Section 4 in the context of SUSY GUTs, and Subsection 5.1 for the impact of possible CP violation in the B_s system.

3.3 Flavour-changing decays: higgs penguins

For weak decays, only one quark flavour needs to be changed, which implies contribution from penguin graphs. (Of course, there are also box graphs.) Higgs penguins generating op-

erators such as

$$\sum_q y_q (\bar{s}_L b_R) (\bar{q}_L q_R)$$

are negligible in the SM, due to the smallness of down-type Yukawa couplings. This continues to hold in the MSSM for small $\tan\beta$. As previously mentioned, at large $\tan\beta$, depending on the Higgs sector this may change. There is a peculiar phenomenology already for minimal flavour violation, see e.g. [17, 18, 19, 61, 62, 63, 96, 97, 98, 99, 100, 101, 102, 103] and [20] for a review; the possible relevance of double Higgs penguins for $B - \bar{B}$ mixing has already been mentioned. For detailed studies of large- $\tan\beta$ effects beyond minimal flavour violation we refer to [93, 104, 105, 106].

3.4 Electroweak penguins

Photon and Z penguins contribute to operators (shown here for $b \rightarrow s$ transitions)

$$Q_{7\gamma} = -\frac{e m_b}{4\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \quad (77)$$

$$Q_{9V} = (\bar{s}_L \gamma^\mu b_L) (\bar{l} \gamma_\mu l), \quad (78)$$

$$Q_{10A} = (\bar{s}_L \gamma^\mu b_L) (\bar{l} \gamma_\mu \gamma_5 l), \quad (79)$$

$$Q_{9,10} = \sum_q \frac{3e_q}{2} (\bar{s}_L \gamma^\mu b_L) (\bar{q}_L \gamma_\mu q_L), \quad (80)$$

$$Q_{7,8} = \sum_q \frac{3e_q}{2} (\bar{s}_L \gamma^\mu b_L) (\bar{q}_R \gamma_\mu q_R), \quad (81)$$

as well as their mirror images Q'_i (obtained by flipping the chiralities of all fermions). $F_{\mu\nu}$ is the electromagnetic field strength. We have suppressed colour structures differentiating between pairs of operators as indicated on the lhs of each line. These operators also have analogs contributing to lepton-flavour-violating $l_i \rightarrow l_j$ transitions.

In the SM, only the electroweak penguin (EWP) operators shown in (77)–(81) receive significant contributions, which are negligible for the “opposite-chirality” operators Q'_i , due to the V-A structure of the charged currents, which are the only source of flavour violation. (This continues to hold in the MSSM for minimal flavour violation.) Identifying contributions to some process from any of the primed operators would constitute a clear signal of new physics. Tree-level W -boson exchange also generates operators

$$Q_{1,2}^u = (\bar{s}_L \gamma^\mu u_L) (\bar{u}_L \gamma_\mu b_L), \quad (82)$$

$$Q_{1,2}^c = (\bar{s}_L \gamma^\mu c_L) (\bar{c}_L \gamma_\mu b_L), \quad (83)$$

which do not receive comparable contributions in the MSSM.

All penguin operators except the magnetic ones also receive contributions from box diagrams; moreover, boxes can contribute to generic four-fermion operators

$$Q = (\bar{s}_{L,R} b_{L,R}) (\bar{q}_{L,R} q_{L,R}).$$

(Here we have suppressed the several possibilities for the Dirac structure.) Note that, in general, boxes are not suppressed with respect to penguin diagrams [1, 57], as all SUSY penguins as well as boxes decouple as M_W^2/M^2 for M large, as discussed before.

3.4.1 Inclusive $B \rightarrow X_s \gamma$

Although loop-induced already in the Standard Model, the inclusive decay $B \rightarrow X_s \gamma$ has a relatively large branching ratio and is well measured, with a world average [83] of

$$BR(\bar{B} \rightarrow X_s \gamma)_{\text{exp}} = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4} \quad (84)$$

(with a 1.6 GeV lower cut on the photon energy). In the SM, the decay amplitude receives its dominant contributions from $W - t$ loops entering through the magnetic operator $Q_{7\gamma}$ and $W - c$ loops entering through loop contractions of the tree operators $Q_{1,2}^c$. Both contributions are of comparable size and opposite in sign. Other operators are subdominant. The corresponding state-of-the-art NNLO-QCD prediction reads [107] (see [108] for a review and discussion of uncertainties)

$$BR(\bar{B} \rightarrow X_s \gamma)_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}, \quad (85)$$

slightly more than 1σ below the experiment. Supersymmetric effects have been investigated thoroughly in the literature both in minimal flavour violation [10, 53, 63, 96, 109, 110, 111, 112, 113, 114] and beyond [1, 51, 52, 115, 116, 117]. They enter mainly through $Q_{7\gamma}$ and the chromomagnetic operator Q_{8g} (defined in (88) below), which has a large mixing with $Q_{7\gamma}$ under renormalization. This results in a large sensitivity to the parameters $(\delta_{sb}^d)_{LR}$ and $(\delta_{ct}^u)_{LR}$, which allows to constrain these parameters, for which we refer to the cited literature and to [2]. Complete expressions for the magnetic operators that are valid for general flavour violation can be found in [115]. See also [118]. Contributions via $Q'_{7\gamma}$ do not interfere with the SM contribution in an inclusive process due to the opposite chirality of the produced s -quark, hence generally have a small impact. The full one-loop SUSY contribution may involve a cancellation between charged-Higgs-top loops, which are always of the same sign as the SM piece, and squark-higgsino as well as squark-gaugino loops, which (in general) carry an arbitrary complex phase.

3.4.2 Z -penguins and rare K, B decays

The decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are almost unique in that they are essentially free of hadronic uncertainties. In the SM context, the two modes provide a clean and independent determination of the unitarity triangle [119, 120, 121, 122, 123, 124, 125]—once they will have been measured precisely, hopefully, at CERN P-326/NA62 (formerly NA48/III) and at JPARC E14. (The SM branching fractions are $\mathcal{O}(10^{-11})$ and $\mathcal{O}(10^{-10})$, respectively.)

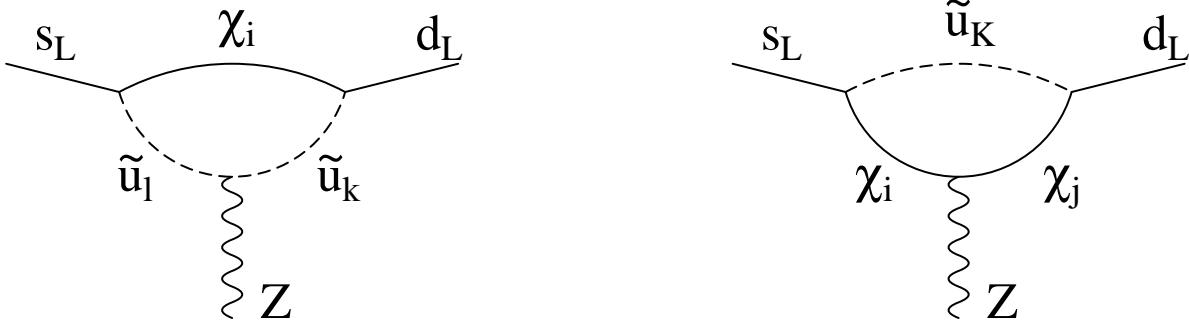


Fig. 4. Potentially large contributions to the Z -penguin (shown here for $s \rightarrow d$ transitions [58])

They are a precise probe the FCNC vertices of the Z boson. In view of the particular, $SU(2)$ -breaking structure of the leading Zsd vertex, this implies a specific sensitivity to certain combinations of LR and RL δ -parameters, even in the presence of general flavour violation [58, 126, 127, 128, 129, 130]. In a perturbative expansion in δ 's,

$$\mathcal{A}(K \rightarrow \pi \nu \bar{\nu})^{\text{SUSY}} \propto (\delta_{ut}^{\tilde{u}})_{LR} (\delta_{tc}^{\tilde{u}})_{RL}. \quad (86)$$

This is an example where the (generically) leading effect arises at second order in the mass insertions. The double LR mass insertion enforces a factor v^2/M^2 required by the decoupling theorem. A systematic numerical analysis [129] (see also [130]) shows that this parametric dependence continues to hold beyond the perturbative expansion, and even in the presence of large contributions from box diagrams (Fig. 5). Moreover the SM hierarchy between the charged and neutral modes may be reversed, the latter enhanced by an order of magnitude or more, and the bound following from isospin symmetry of strong interactions [131] saturated. Indeed, at present the experimental upper bound provides the best constraints on the cited double mass insertion. Complementary probes of the Z -penguin amplitude are provided by the modes $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$, which are still theoretically quite clean [130, 132]. Analogous Z -penguin effects are possible in $b \rightarrow d$ and $b \rightarrow s$ transitions, see the next subsubsection and the brief discussion of EWP effects in Section 5.

3.4.3 Leptonic B decays

Among B -decays, the modes $B^+ \rightarrow \ell^+ \nu$ and $B_d, B_s \rightarrow \ell^+ \ell^-$ are the theoretically cleanest. The former proceeds through a W^+ tree-level diagram in the SM. Significant corrections may occur due to charged-Higgs-boson exchange, which is present in the MSSM [99, 133, 134] but becomes relevant only at large values of $\tan \beta$. In this case, the latter mode can be enhanced by an order of magnitude or more, which provides a serious constraint on certain

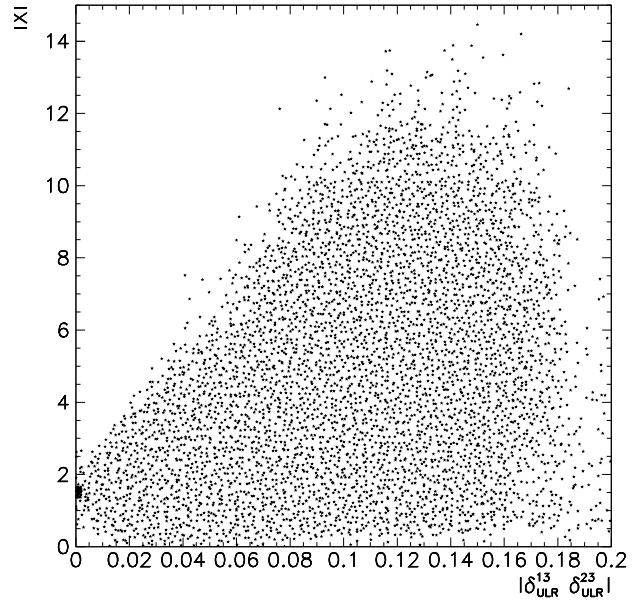


Fig. 5. SUSY contribution X to $\mathcal{A}(K \rightarrow \pi \nu \bar{\nu})$ [129]

large- $\tan \beta$ scenarios. (The branching fraction scales with the sixth power of $\tan \beta$.) We would like to emphasize that at any $\tan \beta$, it will receive more moderate contributions via the Z -penguin, through the operator Q_{10A} , which could still be sizable (compare the discussion in the previous subsubsection). In the SM one has [135]

$$BR(B_s \rightarrow \mu^+ \mu^-) = (3.51 \pm 0.50) \times 10^{-9}, \quad (87)$$

where the bulk of the CKM and hadronic uncertainties has been eliminated by normalizing to ΔM_s . (The error will become even smaller with improved lattice predictions for \hat{B}_{B_s} .) In spite of this mode being so rare, LHCb, ATLAS, and CMS expect to collect a combined few hundred SM events after five years or so of running.

3.4.4 Inclusive $B \rightarrow X_s \ell^+ \ell^-$

Their sensitivity to semileptonic operators like Q_{9V} and Q_{10A} makes the rare $b \rightarrow s \ell^+ \ell^-$ transitions a complementary and more complex test of the underlying theory than the radiative ones. The presence of two charged leptons in the final state allows to define several observables including dependence on the kinematics. In the absence of large statistics, partially integrated spectra such as the dilepton mass spectrum or the angular distribution can be explored that are amenable to a systematic theoretical description for a dilepton invariant mass below the charm resonances.

The decay rate has been considered (in msugra) in [53]. In the (general) minimally flavour-violating MSSM [136, 137, 138] the Wilson coefficients C_{9V} and C_{10A} are only slightly affected and corrections to the decay distributions do not exceed the 30% level.

At large $\tan \beta$, additional contributions to $b \rightarrow s \mu^+ \mu^-$ arise from the chirality-flipping operators $(\bar{s}_L b_R)(\bar{\mu}_L \mu_R)$ and $(\bar{s}_L b_R)(\bar{\mu}_R \mu_L)$ that are suppressed by powers of the muon mass but enhanced by $(\tan \beta)^3$. In practice, these contributions are however bounded from above [139, 140, 141] by the experimental constraints on $B_s \rightarrow \mu^+ \mu^-$ and turn out to be subleading. Merging the information on $B \rightarrow X_s \ell^+ \ell^-$ with the one on $B \rightarrow X_s \gamma$, one can thus infer that the sign of the $b \rightarrow s \gamma$ amplitude should be SM-like [142] in MFV. Expressions to the semileptonic Wilson coefficients relevant for the general MSSM can be found in [118]. See also [143]. A simultaneous use of the $B \rightarrow X_s \ell^+ \ell^-$ and $B \rightarrow X_s \gamma$ constraints then leads to stringent limits on $(\delta_{sb}^d)_{LL}$ and $(\delta_{sb}^d)_{LR}$ in the complex plane [117, 144].

3.4.5 Exclusive semileptonic and radiative B -decays

Exclusive semileptonic and radiative B -decay modes such as $B \rightarrow K^*(l^+ l^-)$, $B \rightarrow K^* \gamma$, etc. are also natural as a place to look for physics beyond the Standard Model; they will be (much) more readily accessible than the inclusive ones in a hadronic environment such as LHCb. However, the theoretical treatment is somewhat more involved and they have not received as much attention in a supersymmetric context as their inclusive counterparts. See e.g. [145] for a proposal to look for supersymmetric right-handed currents in $B \rightarrow K^* l^+ l^-$. More work on SUSY effects in these observables would be desirable.

3.5 Combined constraints

The constraints on the flavour-violating parameters become more powerful when the interplay of several observables with different parametric sensitivities is considered. (This was considered in a number of the works referred to above.) For $b \rightarrow s$ transitions, combined constraints on the δ_{sb} parameters were obtained and graphically displayed in ref. [144]. Fig. 6 [144] shows how the measurements of from $B \rightarrow X_s \gamma$, $B \rightarrow X_s \ell^+ \ell^-$, and ΔM_s coact to constrain the parameter $(\delta_{sb}^d)_{LL}$, leaving a much smaller allowed region than each individual observable.

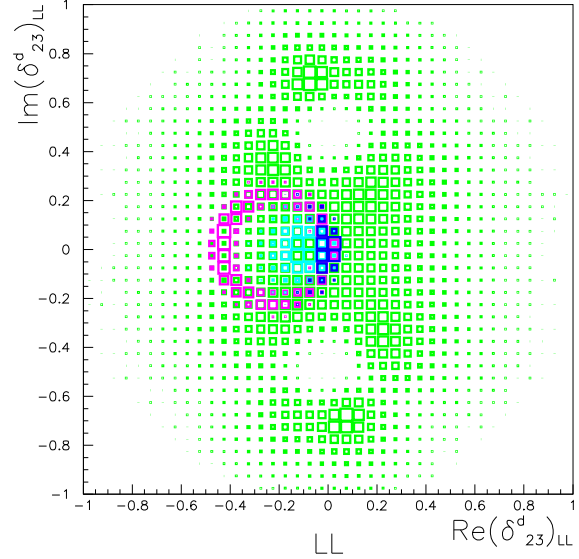


Fig. 6. Combined constraints on $(\delta_{sb}^d)_{LL}$ [144]. The allowed regions due to $B \rightarrow X_s \gamma$, $B \rightarrow X_s \ell^+ \ell^-$, and $B_s - \bar{B}_s$ mixing are shown in violet, light blue, and green, respectively; the combined constraint is in dark blue. A common particle mass of 350 GeV and $\mu = -350$ GeV is assumed.

3.6 QCD penguins

QCD-penguin graphs contribute only to $\Delta F = 1$ transitions via the operators

$$Q_{8g} = -\frac{g_s m_b}{4\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R G_{\mu\nu}, \quad (88)$$

$$Q_{3,4} = \sum_q (\bar{s}_L \gamma^\mu b_L) (\bar{q}_L \gamma_\mu q_L), \quad (89)$$

$$Q_{5,6} = \sum_q (\bar{s}_L \gamma^\mu b_L) (\bar{q}_R \gamma_\mu q_R), \quad (90)$$

together with operators Q'_i which arise from the Q_i by changing the chiralities of all quarks. Here we have taken the case of $b \rightarrow s$ transitions as an example, with obvious replacements for $b \rightarrow d$ and $d \rightarrow s$ transitions. We have also suppressed colour and part of the Dirac structure. (G is the gluon field strength.) Note that the QCD penguins $Q_{3...6}^{(\prime)}$ involve b and s quarks of like chiralities, while the chromomagnetic penguins $Q_{8g}^{(\prime)}$ involve a chirality flip. As with the electroweak penguins, this engenders specific patterns of sensitivity of their coefficients to the parameters δ , e.g. $(\delta_{sb}^d)_{LR}$ in the case of C_{8g} .

3.6.1 Charmless hadronic decays

Two-body exclusive nonleptonic decays $B \rightarrow M_1 M_2$ are sensitive to all of the operators $Q_{1...10,7\gamma,8g}^{(\prime)}$, while offering

a large number $\mathcal{O}(100)$ of observables, including many CP-violating ones. They (and exclusive modes in general) will increase in importance due to their accessibility at hadron machines such as the LHC. On the theoretical side, the factor limiting the precision are the hadronic matrix elements $\langle M_1 M_2 | Q_i | B \rangle$, which involve nonperturbative QCD in a way that is presently not surmountable in lattice QCD. Systematic methods are, however, available, based on expansions about the limit of $SU(3)$ flavour symmetry or about the heavy- b -quark limit. m_s/Λ and Λ/m_b , respectively, are the expansion parameters. In fact, the (QCD) factorization formulae [146, 147, 148] for the hadronic matrix elements that follow from the heavy-quark limit respect the $SU(3)$ flavour symmetry up to well-defined (so-called “factorizable”) corrections at the leading power and perturbative order, and perturbative QCD corrections do not alter this picture very much. Higher orders in Λ/m_b are generally not under control, with certain (important) exceptions. We cannot go into more detail here but refer to the brief discussion of $b \rightarrow s$ penguin modes in Section 5.

3.7 Lepton flavour violation

It is beyond the scope of this article to cover lepton flavour violation in detail. In the SM, even when introducing the minimal dimension-five operator to allow for neutrino masses and mixings, lepton-flavour-violating processes such as $\tau \rightarrow \mu\gamma$ are rendered extremely rare by the tiny neutrino mass splittings and the corresponding near-perfect GIM cancellation. The situation is very different in the MSSM because of the presence of lepton flavour violation at the renormalizable level, in the sneutrino and charged slepton mass matrices. Nonobservation of SUSY effects in $\ell_i \rightarrow \ell_j\gamma$ leads to stringent bounds on sleptonic mass insertions [1], ruling out generic flavour structures for them. Predictions for MSSM (charged) lepton flavour violation are often considered in seesaw scenarios. See [149] for a recent review, and Section 4

3.8 Implications of low-energy observables for collider-physics measurements

The flavour-violating parameters that govern low-energy observables also affect production cross sections for sparticles, and their decays; a detailed investigation would go beyond the scope of the present review. For recent works in that direction, see e.g. [150, 151, 152, 153, 154, 155], and [156] (which also deals with many of the flavour observables discussed above) and references therein.

4 Probing the GUT scale

Concrete assumptions about the SUSY-breaking mechanism (gravity mediation, gauge mediation, etc.) and possible UV completion (such as a SUSY grand-unified theory, minimal flavour violation, etc.) may imply patterns

in $\mathcal{L}_{\text{soft}}$ relating different δ parameters that can be tested against the general constraints applying to them, or can be further used in making specific predictions for low-energy observables and their correlations. We do not discuss specific SUSY models in this article, referring instead to other contributions to this volume [157, 158] for the status of SUSY model building, but consider only a very minimal GUT setup for illustration. One of the most intriguing aspects of SUSY GUTs, which also demonstrates the power of flavour observables to probe even superhigh scales, is the possibility of relations between hadronic and leptonic flavour violation [159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181]. The effect we are considering here is the following [12, 160, 161]. Assume that SUSY breaking is effected at a scale beyond the GUT scale, for instance at the Planck scale, and that it is flavour-blind, at least approximately, such that one has a universal scalar mass parameter m_0^2 and a universal trilinear scalar coupling parameter a_0 . For definiteness, assume simple $SO(10)$ unification such that there is only one sfermion multiplet for each generation. Radiative corrections due to the unified gauge coupling will correct the masses of the three 16’s of sfermions in the same way, while the large top Yukawa coupling will selectively suppress the masses of one multiplet,

$$m_{16_{1,2}}^2 = m_0^2 + \epsilon, \quad m_{16_3}^2 = m_0^2 + \epsilon - \Delta, \quad (91)$$

where $\epsilon \propto g^2$ and $\Delta \propto y_t^2 m_0^2$, $y_t^2 a_0^2$. Eq. (91) holds in a basis where the up-type Yukawa matrix is diagonal. (Further contributions will be present if there are additional large Yukawa couplings, such as for large $\tan\beta$. The expressions then become more complicated.) In this fashion, the large top Yukawa coupling affects also the right-handed down-type squark masses and the slepton masses, which is very different from the situation in the MSSM (or below the GUT scale). Now the relevant sfermion basis for low-energy physics (the super-CKM basis) is the one where the down-type and leptonic Yukawas are diagonal. In such a basis the members of the 16₃ selected by y_t will consist of mixtures of the superpartners of fermions of different flavours, and FCNC SUSY vertices appear. Moreover, these vertices will be correlated between the hadronic and leptonic sectors. This kind of mechanism received renewed attraction after the observation of large leptonic mixing in neutrino oscillations [167]. Assuming a minimal- $SU(5)$ -type embedding of the MSSM into $SO(10)$, and under certain assumptions about the generation of seesaw neutrino masses, one has

$$Y_D = U_{\text{PMNS}}^T \text{diag}(y_d, y_s, y_b) V_{\text{CKM}}^\dagger. \quad (92)$$

This expression should be most robust in the (2, 3) subblock. The atmospheric neutrino mixing angle then appears in the gluino-squark couplings

$$\mathcal{L}_{\text{soft}} \supset U_{\text{PMNS}}^{ij} \tilde{d}_{Rj}^* \tilde{g} T^A d_{Ri}, \quad (93)$$

yielding potentially spectacular effects in observables like ΔM_{B_s} [172, 173, 174, 175]. The model is parameterized by

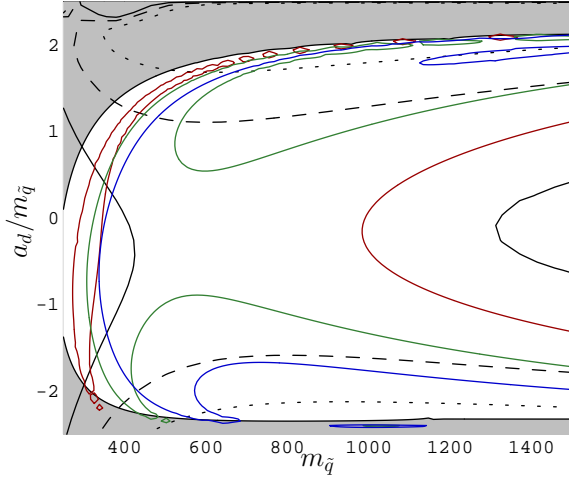


Fig. 7. Contours of constant SUSY contribution to ΔM_{B_s} , in units of the SM value, and of constant $BR(\tau \rightarrow \mu\gamma)$, in a slice of parameter space [175]. Here a_d has a simple relation to a_0 , $m_{\tilde{g}} = 250$ GeV, $\tan\beta = 3$. The solid black, dashed, and dotted contours correspond to $|\Delta M_{B_s}^{(NP)}|/\Delta M_{B_s}^{(SM)} = 0.5, 2, 5$, respectively. The red, green, and blue contours correspond to the experimental upper bound on $BR(\tau \rightarrow \mu\gamma)$ for $\mu = -300$, $\mu = -450$, $\mu = -600$ GeV, respectively. (ΔM_{B_s} is independent of μ in the approximation used.)

four parameters m_0 , a_0 , $m_{\tilde{g}}$, and μ , and a value of $\tan\beta$ around 2–3 to maintain perturbativity. Fig. 7 compares $BR(\tau \rightarrow \mu\gamma)$ with the correction to ΔM_{B_s} . It is evident that the measurement of ΔM_{B_s} provides a nontrivial and quantifiable constraint on the lepton-flavour-violating mode, providing (one of many) illustration(s) of the possibility to probe very fundamental scales with flavour-violating observables, beyond what is possible with knowing the particle spectrum alone.

5 Hints of departures from the standard model

5.1 CP violation in $B_s - \bar{B}_s$ mixing?

We mentioned in Section 3.2.3 that the Standard Model predicts, in a theoretically very clean manner, negligible mixing-induced CP violation in B_s decays, and also the Tevatron measurements of time-dependent CP violation in $B \rightarrow J/\psi\phi$, which after 2008 summer conferences shows an (increased) 2.2σ discrepancy with the SM expectation. In fact, as mentioned in Section 3.2.3, further information on ϕ_s can be inferred from the “semileptonic CP asymmetry” or similar asymmetries in other flavour-specific B_s decays. Combined fits have been performed by the UTfit and CKMfitter collaborations. This raises the discrepancies with the SM in ϕ_s to 2.5 and 2.4σ [182, 183, 184], respectively. Recall that the expected precision after 5 years of LHCb is about 1 degree, hence this has the potential to turn into a very significant falsification of the SM. It is not

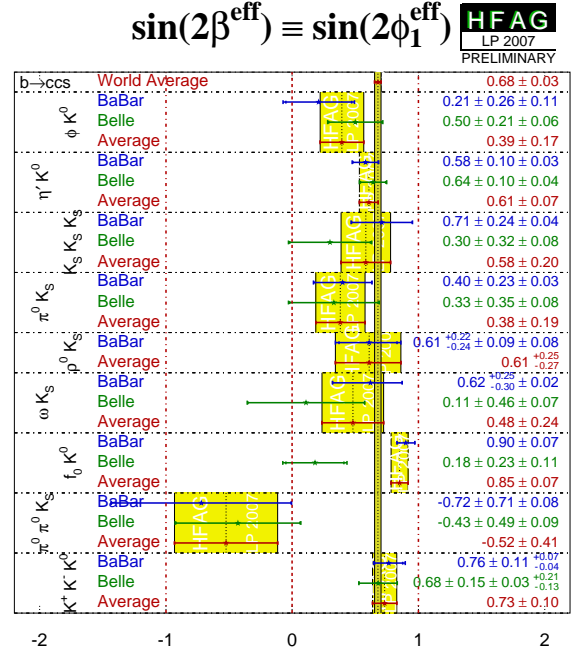


Fig. 8. Mixing-induced CP violation in charmless $b \rightarrow s$ penguin transitions [83].

difficult to generate such a signal in the MSSM, as $b \rightarrow s$ FC parameters are not very strongly constrained [179, 180, 181]. These papers also relate the effects in $B_s - \bar{B}_s$ mixing to the branching fraction for $\tau \rightarrow \mu\gamma$ in the context of SUSY GUTs of the type discussed in Section 4.

5.2 Time-dependent CP asymmetries (“ $\sin 2\beta_{\text{eff}}$ ”)

The most long-standing pattern of observed deviations in flavour physics concerns the closely related class of time-dependent CP asymmetries in $b \rightarrow s$ penguin decays of B_d mesons into CP eigenstates,

$$\frac{BR(\bar{B}^0(t) \rightarrow f) - BR(B^0(t) \rightarrow f)}{BR(\bar{B}^0(t) \rightarrow f) + BR(B^0(t) \rightarrow f)} \equiv S_f \sin(\Delta m_B t) - C_f \cos(\Delta m_B t). \quad (94)$$

Within the SM one expects $\eta_f S_f \approx \phi_d = 2\beta$, based on the dominance of the QCD penguin amplitude, which has vanishing weak phase. (η_f denotes the CP quantum number of the final state.) Neither equality holds in the MSSM (beyond minimal flavour violation). Fig. 8 shows the data for a number of modes. It is conspicuous that in general, the $\eta_f S_f \equiv \sin(2\beta_{\text{eff}})(f)$ lies below the value of $\sin 2\beta$. This has received much interest in recent years. Unfortunately, these modes are theoretically less clean for new-physics searches, as they require the calculation of at least partial information about hadronic decay amplitudes (at the minimum, restricting a strong phase to a half-plane). Moreover, the significance of the “signal” has gone down over

the past few years. Perhaps we are observing merely a statistical fluctuation. Nevertheless, any reasonably SUSY model that can accommodate CP violation in $B_s - \bar{B}_s$ mixing ($\Delta B = \Delta S = 2$) should give rise to *some* contribution to ($\Delta B = \Delta S = 1$) $b \rightarrow s$ penguin transitions, since the fundamental FC vertices in the MSSM all violate flavour numbers by one.

In fact, in the SM a QCD factorization calculation [185] of the corrections for the two-body final states $\phi K^0, \eta' K^0, \pi^0 K_S, \rho^0 K_S, \omega K_S$ due to neglected smaller amplitudes shows a (small) positive shift in all cases except $\rho^0 K_S$. Supersymmetric contributions to $b \rightarrow s$ hadronic penguin transitions have received considerable attention in the past [115, 117, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 172, 198, 199, 200, 201, 202], under various assumptions about where flavour is violated. Often the most important operators are the magnetic-penguin operators Q_{8g} or Q'_{8g} , which are sensitive to LR δ -parameters. (Q'_{8g} will interfere with the SM contributions in exclusive processes, unlike in inclusive $B \rightarrow X_s \gamma$ decay, which makes it less constrained but possibly relevant.) Indeed, an interesting possibility is to attribute the pattern of the deviations to constructive and destructive interference between operators of different quark chiralities, depending on the parities of the final-state particles [194]. Another possibility to change the $b \rightarrow s$ penguin pattern from its SM form is a modified electroweak penguin, see below.

5.3 Pattern of CP asymmetries in $B \rightarrow \pi K$

$B \rightarrow \pi K$ decays are those charmless hadronic decays that have been studied most with regard to possible new-physics effects. (This set of observables overlaps with those of the previous subsection in the mixing-induced CP violation parameter $S_{\pi^0 K_S}$.) This interest relates to the fact that they are penguin-dominated, receiving important contributions from QCD penguins, as well as electroweak-penguin contributions that one can try to disentangle using an isospin analysis. Also here, some knowledge of hadronic amplitude ratios is needed. Three “puzzles” have been identified.

5.3.1 Ratios of isospin-related branching fractions

While the deviation of the pattern of branching fractions from SM expectations observed in [203, 204, 205, 206] has gone away as the experimental data (and its treatment) have evolved, the puzzle invoked a number of theoretical calculations on the MSSM impact, see in particular [201], as well as the references in the previous subsection, which may be relevant to the next two issues.

5.3.2 Patterns in direct CP asymmetries

Theoretical arguments (such as calculations based on the heavy-quark limit) suggest that

$$\begin{aligned} C_{\pi^- K^+} &\equiv -A_{CP}(B_d^0 \rightarrow \pi^- K^+) \\ &\approx C_{\pi^0 K^+} \equiv -A_{CP}(B^+ \rightarrow \pi^0 K^+) \end{aligned} \quad (95)$$

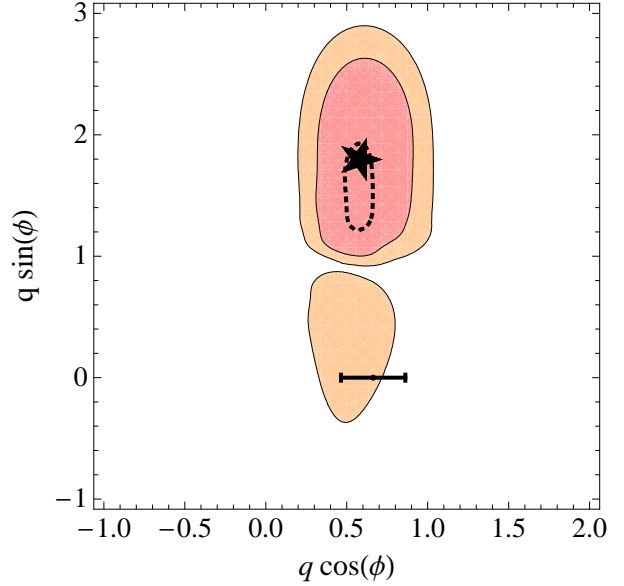


Fig. 9. Electroweak-penguin-to-tree ratio fitted from data [209]. Shown are 1σ and 90 % CL regions. The star denotes the best fit, the bar the SM expected range, which lies along the real line. For details, see [209].

in the Standard Model. The HFAG-averaged data due to Babar, Belle, CDF and CLEO is [83]

$$C_{\pi^- K^+} = 0.097 \pm 0.0012, \quad C_{K^+ \pi^0} = -0.050 \pm 0.025,$$

a $> 5 \sigma$ difference [207, 208]. The theory expectation (in the SM) is hard to quantify precisely, mainly due to the uncertain colour-suppressed tree contribution, but 0.15 appears very large. One mode depends on electroweak penguins but not the other, which makes the difference sensitive to possible, CP-violating, new-physics effects.

5.3.3 Non-standard electroweak penguin and $S_{\pi^0 K_S}$

An alternate route is to use SM isospin relations and use the data on the rates to predict the parameter $S_{\pi^0 K_S}$. This allows using theory input in a more limited fashion. Fig. 9 shows the result of a χ^2 fit of the relevant electroweak penguin parameter to data [209]. Also here, the significance is not very strong. Again, however, the data are certainly consistent with extra SUSY contributions, and we expect on general grounds some correlation with any new-physics effects in $B_s - \bar{B}_s$ mixing.

6 Conclusion

The MSSM contains a large, $\mathcal{O}(100)$, number of new flavour and CP parameters. We have emphasized their tight connection with the mechanism of supersymmetry breaking. The MSSM flavour structures have to be quite non-generic both in the quark and in the lepton sector. However, there is room for deviations from minimal flavour violation. In

fact, several possible hints of NP are present in the data. In the next few years, only LHC can tell us whether they are real or fluctuations.

As this is being written, CERN has just shot its first test bunches into the LHC ring. We may hope that soon the era of putting bounds and constraints will give way to a phase of actual measurements of MSSM parameters.

If SUSY is found, the interplay between direct and indirect observables is likely to be useful, as it was in the construction of the SM. A super- B -factory will be of great help here. Moreover, the peculiar gentleness of SUSY quantum corrections may mean that in SUSY, the GUT or Planck scales can actually be “close” as far as indirect observables are considered (as exemplified in certain SUSY GUTs), even if being at great distance from the point of view of direct detection.

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A Loop functions

$$S(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3} \quad (96)$$

$$f_6(x) = \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5} \quad (97)$$

$$\tilde{f}_6(x) = \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5} \quad (98)$$

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